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Charles Frederick Haberly
Iowa State University

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USING A POWER TRANSMISSION LINE AS A DIGITAL COMMUNICATION CHANNEL IN THE 70 TO 110 CPS RANGE

by

Charles Frederick Haberly

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Head of Major Department

Signature was redacted for privacy.

Dean of Graduate Faculty

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INTRODUCTION AND REVIEW OF PRESENT TECHNIQUES FOR COMMUNICATING

One of the basic criteria of the performance of any communication or signaling circuit is its signal-to-noise ratio (1). In a digital communication channel, poor signal-to-noise ratio may result in unsatisfactory service of a degree varying from barely perceptible background noise to complete masking and total loss of information. The general term "noise" is used in this paper to designate all electrical disturbances that may give rise to unwanted frequency components within the acceptance band of a receiver, exclusive of intentionally generated continuous wave signals.

The purpose of this dissertation is to report on the results of one investigation being conducted in cooperation with the Affiliate Research Program in Electric Power at Iowa State University, Ames, Iowa. This study was conducted with the specific notion of making rms noise voltage and current measurements on a 60 cps transmission line and using it as a digital communication channel. These measurements show the actual amount of noise on the line from 70 to 110 cps. Using these measurements, it is possible to find the power per bandwidth versus frequency for any voltage or current in the 70 to 110 cps range. Once the noise power per bandwidth is known, it is possible to predict the amount of signal power that is needed to communicate from one location to another over the 60 cps transmission line.

The experimental results for both the rms noise voltage and current versus frequency are presented. The equipment used, its characteristics, and the method used to obtain these measurements are also presented. A mathematical model of the rms noise voltage or current versus frequency was developed and was used to predict the rms noise voltage or current on any line with the average voltage or current known. It was assumed that all the noise was completely random in the development.

The problem of communicating from a remote location to the source is treated first. The theory for a single bit of information is developed. A single information bit is generated by connecting a load to the line for two cycles and then disconnecting it for two cycles and repeating this for several cycles. The Fourier transform of the actual signal is divided into two equal parts that are centered at 60 and -60 cps.

A finite-time spectral density function and the interpretation of this function for a finite time function is developed. The normal power spectral density function is independent of time, but the finite-time spectral density function developed in this paper is time dependent. Several finite-time spectral density functions are plotted versus frequency to show the effect of increasing the length of a single information bit. The power contained in a given bandwidth is found. Using the experimental results for current noise power and the above results, it is possible to predict the amount of current that the load must draw to communicate with the source for any signal-to-noise ratio.

The second problem treated is that of communicating from the source to any remote location. A single information bit is generated by coupling to the line a sinusoidal signal in the 70 to 110 cps range for a finite time. The Fourier transform and the finite-time spectral density functions of this single information bit are found. Several finite-time spectral density functions are plotted versus frequency to show the effect of increasing the length of a single information bit. The power contained in a given bandwidth is found. Using the experimental results for voltage noise power and the above results it is possible to predict the amount of power that must be supplied at the source to communicate with any distant location for any signal-to-noise ratio.

Modern power system operation has greatly increased the need for intrasystem and intersystem communications. In its broadest sense the word communication must include the interchange of many types of intelligence such as voice, teletype, supervisory control, telemetering, protective relaying, and load control (2).

At the present time, the following methods are in common use (3):

1. Microwave utilizing transmitters and receivers operating in the 1700 to 6800 megacycle frequency band.
2. Power-line carrier utilizing transmitters and receivers operating in the 30 to 200 kilocycle frequency band, and using the power transmission line as the propagation medium for these signals.
3. Audio tone equipment using transmitters and receivers operating in the 600 to 3000 cycle frequency band, utilizing a privately owned or leased pilot wire channel, or a modulation on microwave channels.

4. Pilot-wire equipment using either 60 cps power frequency or d-c signals.

Blackburn and Rockefeller (4) state that microwave is limited to key areas of the system where many functions ride the microwave circuits. Where ten or more channels are required between any two points, microwave is economically feasible. They also compare the choice between a pilot-wire and power-line carrier. They state that the cost of coupling radio-frequency energy to a power line must be balanced against the cost of providing a direct wire connection. Unless the pilot-wire cable cost is shared by several functions, a length of 7 to 10 miles represents the economic limit.

Cheek (5) reports on the limitations of the frequency-modulation system for power-line carrier work, and furnishes a comparison of the amplitude-modulation, frequency-modulation, and single-side-band systems on the basis of their channel-width requirements, their ability to work through attenuation, and their susceptibility to interference and noise. He shows that the single-side-band system with suppressed carrier has an advantage in signal-to-noise ratio and uses only one-half the frequency spectrum.

Lenehan (6) reports on a new single-side-band carrier system for power lines. He states the need to conserve the frequency spectrum and suggests that a single-side-band signal be used and discusses a method of generating the signal.

Rives (7) reports on the application of carrier to power lines. He discusses the problem of coupling the carriers to the line, line-tuning equipment, and line traps. The carrier is coupled to the line using a high voltage capacitor. Line-tuning equipment is used for impedance levels that are large for the power line yet allow the carrier frequency to pass through with very little loss. The line traps are placed in series with the line to allow the power frequency to pass but represent a high impedance for the carrier frequency. He also discusses coupling around large transformer banks and carrier-circuit attenuation.

The AIEE Committee report (8) gives a survey to present data which can be used to indicate the extent to which pilot-wire relaying is used, the types of protective devices used for the pilot wires, how the pilot wires are supervised, the general types of pilot-wire relaying used, the extent to which fault detectors are used, the outages and per-cent availability of pilot wires, and the operating record of the pilot-wire relays.

A search of the literature has failed to reveal any information on the noise on a 60 cps transmission line in the 70 to 110 cps range. It was decided to investigate the noise level on a 60 cps line. As the investigation was to be conducted on the Iowa State University campus the power plant which supplies power to the university was an ideal choice. The power plant is a three-phase grounded wye-wye system. The steam driven generator has 4160 volts phase-to-phase and supplies a peak power of four megawatts.

BRIEF INTRODUCTION TO PROPOSED TECHNIQUE FOR SIGNALING IN THE 70-110 CPS RANGE

The reason for choosing the frequency range between 70 and 110 cps is because no other equipment is needed at the transformer banks in a power distribution system. The attenuation of the signal in the 70 to 110 cps range due to the transformer banks is in the same order of magnitude as the attenuation of the 60 cps carrier. The only equipment needed to communicate from one location to the other is the terminal equipment or a transmitter at one end and a receiver at the other end. The signal in the 70 to 110 cps range is not affected by the line-tuning, line traps, and the coupling equipment on systems that use a power-line carrier as a communication channel.

To communicate from any location in the field back to the source it is necessary to place information on the line at a certain frequency and at a given rate so that it can be decoded at the source. If a load is added to the line in a periodic manner certain frequency components will be present in the current i(t) drawn by this load. The load must be added to and taken off the line at precise times in order to generate the proper frequency components in the signal that is sent to the source. If, when the voltages are exactly zero and going positive, loads are added to and taken off the line for exactly two complete cycles the frequency components in the signal are predictable. If a load is placed on the line for exactly two cycles and taken off for two cycles and repeated for a long time its frequency components can be found from a Fourier series of i(t).

When a load is placed on the line at a remote location for precisely T_i seconds and removed from the line for precisely T_j seconds and repeated N times the current drawn by the load will have predictable frequency components in it. If the sum of T_i plus T_j is T_k the first useable frequency component for communicating with the source is $(60 + 1/T_k)$ cps. The communication channel is limited to the 70 to 110 cps range so the value of T_k is between 1/50 and 1/10 seconds. The values of T_i and T_j need not be the same. If the zero crossing of the 60 cps voltage is used as a timing reference and if T_k is an integer of 1/120 seconds the maximum time T_k could have is 12/120 seconds and the minimum time T_k could have is 3/120 seconds. This assures that the first useable frequency component for communicating with the source is in the 70 to 110 cps range. By properly adjusting the switching time of the load, R, the current i(t) is made up of different frequency components. Once the maximum current, I, from the source is known it is possible to predict the value of R so that the frequency components of i(t) are larger than the noise components contained in a given bandwidth. The physical location of R is independent of the system, but its value depends on the system in which it is used.

EXPERIMENTAL INVESTIGATION OF NOISE ON 60 CPS TRANSMISSION LINE

Actual Noise Measurements

Voltage

The equipment shown in Fig. 1 was used to obtain the rms noise voltage variations on a 120 volt 60 cps line over the frequency range from 60 to 120 cps. The twin-T network was used to block the 60 cps component. A plot of E_{out}/E_{in} versus frequency for the twin-T network can be found in Fig. 2. The Hewlett-Packard (HP) 302A wave analyzer has a seven cycle bandwidth and was used to obtain a small portion of the frequency spectrum. This output was mixed with an oscillator which translated the output frequencies of the HP 302A. The output from the mixer circuit was amplified and used to drive a mechanical filter. The mechanical filter was a 440 cps tuning fork which has a bandwidth of about one-half cycle. The output of the tuning fork was proportional to the rms noise voltage contained in a one-half cycle bandwidth on the 120 volt line. This output was fed into a HP 3400 true reading rms voltmeter. The average readingsof the HP 3400 was recorded and from a calibration curve of the equipment it was possible to find the actual rms noise readings on the line. If the bandwidth characteristics of the HP 302A and the tuning fork were ideal these readings would be exact, but there were errors due to the fact that they were not ideal.

The incoming signal into the HP 302A wave analyzer was first attenuated to a level which prevents overloading the input amplifier. After the signal was amplified, a low-pass filter rejected any possible 100 kc component. Frequency conversion of the input signal to the 100 kc intermediate frequency occurs in the balanced modulator, which was driven by a 100 to 150 kc local oscillator. The high selectivity of the instrument was obtained by cascading two crystal-tuned IF filters. The local oscillator signal was mixed with the 100 kc IF signal in the output modulator. The lower sideband component was identical with the original input signal frequency and was available at the output jack. It was possible to obtain the bandwidth characteristics shown in Fig. 3 for the HP 302A over its operating range from 20 to 50,000 cps. The output from the HP 302A includes only frequency components contained in the bandwidth of the instrument.

Data for the bandwidth characteristics of the tuning fork, with a Q of about 880, was taken and plotted in Fig. 4. The bandwidth was 1, 2, and 4 cycles when the output voltage was 15, 20.6, and 25.8 db down respectively. A small coil was used to drive the tuning fork and a second coil with a direct-current winding was used to detect the signal.

To calibrate the amplifier and mixer, tuning fork, and rms voltmeter a substitution method was used. A pure 90 cps tone of known amplitude was fed into the amplifier and mixer. It should be noted that under test conditions the location of this 90 cps signal was actually the output of the HP 302A and it was called E_T (Fig. 1). This 90 cps was translated in frequency by the mixer and was used to drive the mechanical filter. The output of the funing fork was recorded on the rms voltmeter. By changing the input amplitude a curve for the input versus output of these three instruments was obtained. For a given reading on the rms voltmeter this curve was used to find the corresponding reading for the input voltage. A calibration curve was generated before any actual readings were taken. It should be noted that this calibration curve gave the actual rms voltage into the amplifier and mixer circuits (E_T).

The input to the HP 302A under actual test conditions was found once E_T was known. The instrument was calibrated in 10 db steps from 30 microvolts to 300 volts. Each scale was linear so the actual input to the HP 302A was determined once E_T was known. The input to the HP 302A was called E_A (Fig. 1). Knowing the value of E_A it was possible to use Fig. 2 to determine the actual voltage on the 120 volt line for each observed frequency.

The actual measurements were taken in two cycle intervals from 60 to 120 cps. The HP 302A bandwidth was centered about the frequency of interest. A frequency counter was used to determine the exact frequency of the HP 302A so that it was centered properly. The output, E_T, from the HP 302A contained only the frequency components within its bandwidth. E_T was mixed with a local oscillator which was tuned to exactly the proper frequency for the tuning fork. A frequency counter was also used to check the frequency of the local oscillator. The output from the tuning fork contained the actual frequency components in its very small bandwidth and was read by the HP 3400 true reading rms voltmeter. Four readings were taken at each setting and the average of these readings was recorded. A period of about two hours was required to obtain a complete set of readings from 60 to 120 cps. Readings were recorded continuously until a total of ten sets were taken (Fig. 5). Table 1 shows E_A, the average voltage, into the HP 302A for each observed frequency. The values of E_{out}/E_{in} were obtained from Fig. 2. The constant, Mult., in Table 1 was used to find V_L once the decibel ratio of E_{out}/E_{in} was known from Fig. 2. The values of V_L were the product of E_A and Mult.

To correct the rms noise voltage versus frequency curve in the vicinity of 60 and 120 cps it was necessary to look at the sum of the HP 302A and the tuning fork bandwidth characteristics (Fig. 6). The points of interest were $f_r \pm 2$, $f_r \pm 4$, $f_r \pm 6$, $f_r \pm 8$, and $f_r \pm 10$ cps. It was known that when a reading was taken at 62, 64, 66, 68, and 70 cps or 118, 116, 114, 112, and 110 cps that a certain amount of the 60 or 120 cps signal was being added to this reading. The problem was to predict the amount of the 60 or 120 cps signal from the composite bandwidth characteristic and subtract this quantity from the actual measurement in order to obtain the corrected reading. The only readings that were corrected were in the ranges from 60 to 70 cps and 110 to 120 cps. These ranges did have a large error because of the large 60 and 120 cps components.

If an ideal, noiseless, 120 volt 60 cps line were available it would be possible to connect the twin-T filter and the composite bandwidth characteristic, shown in Fig. 6, to it and measure the amount of 60 cps voltage at each frequency of interest. The amount of 60 cps voltage

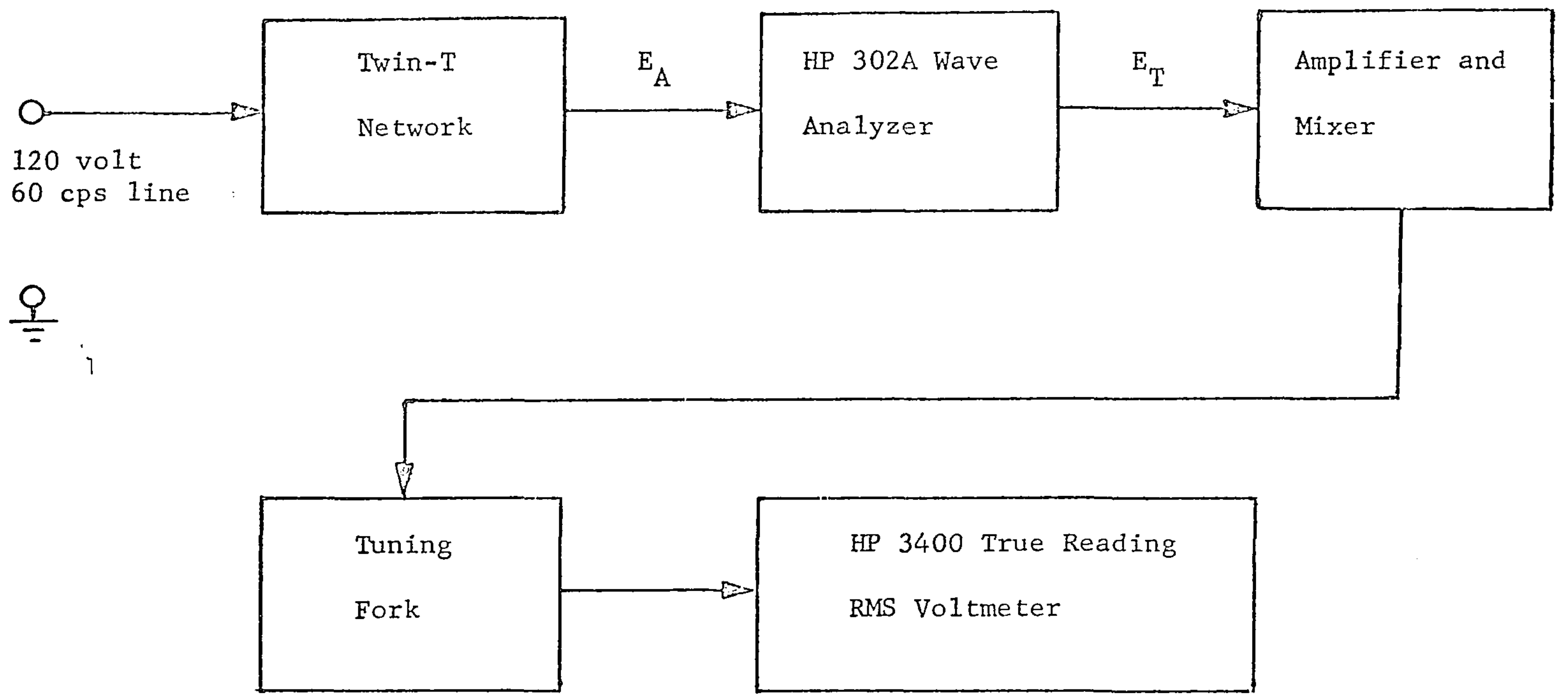


Fig. 1. Block diagram for measuring the rms noise voltage on a 60 cps line

Fig. 2. Measured E_{out}/E_{in} versus frequency for twin-T network

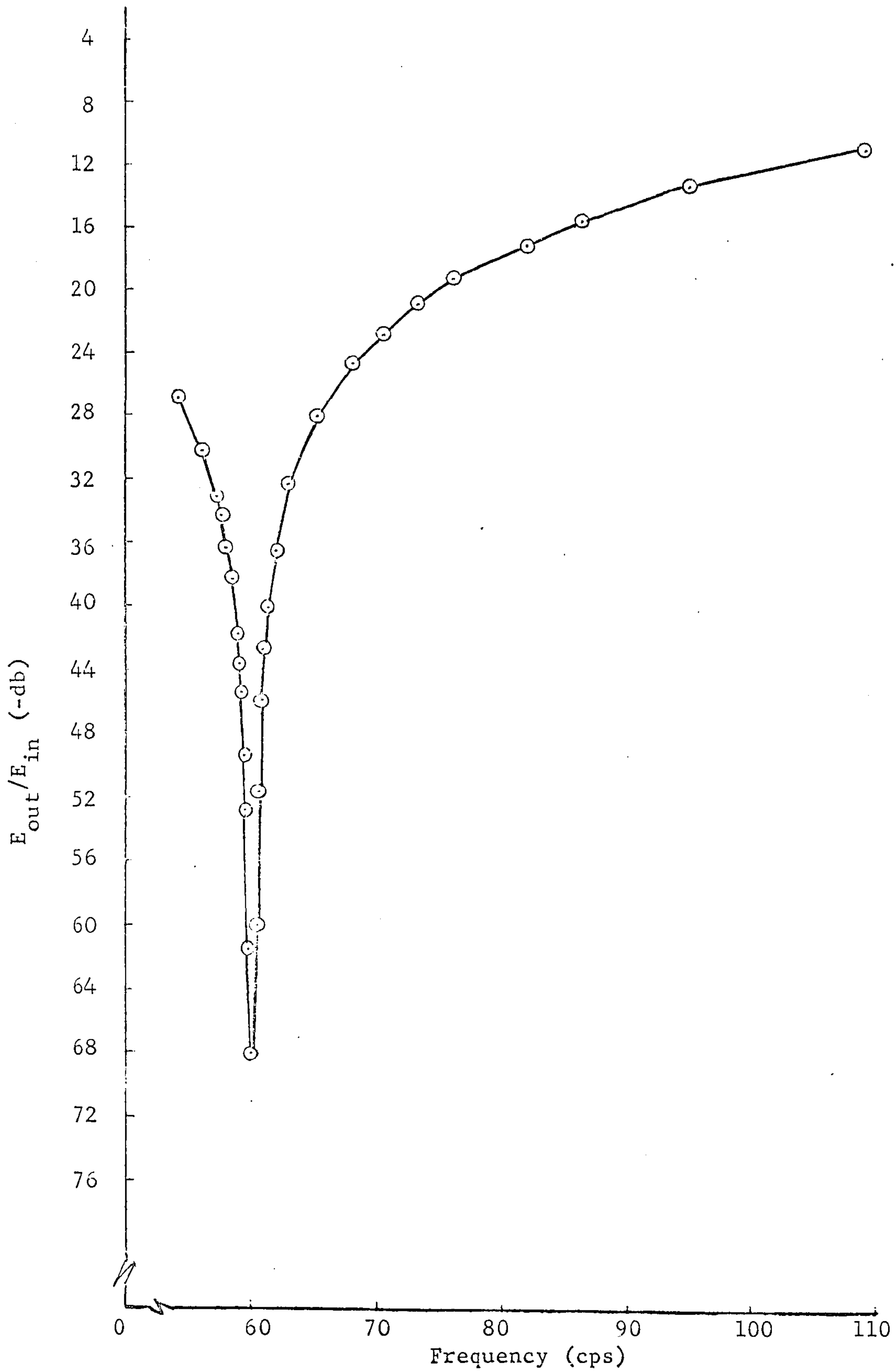
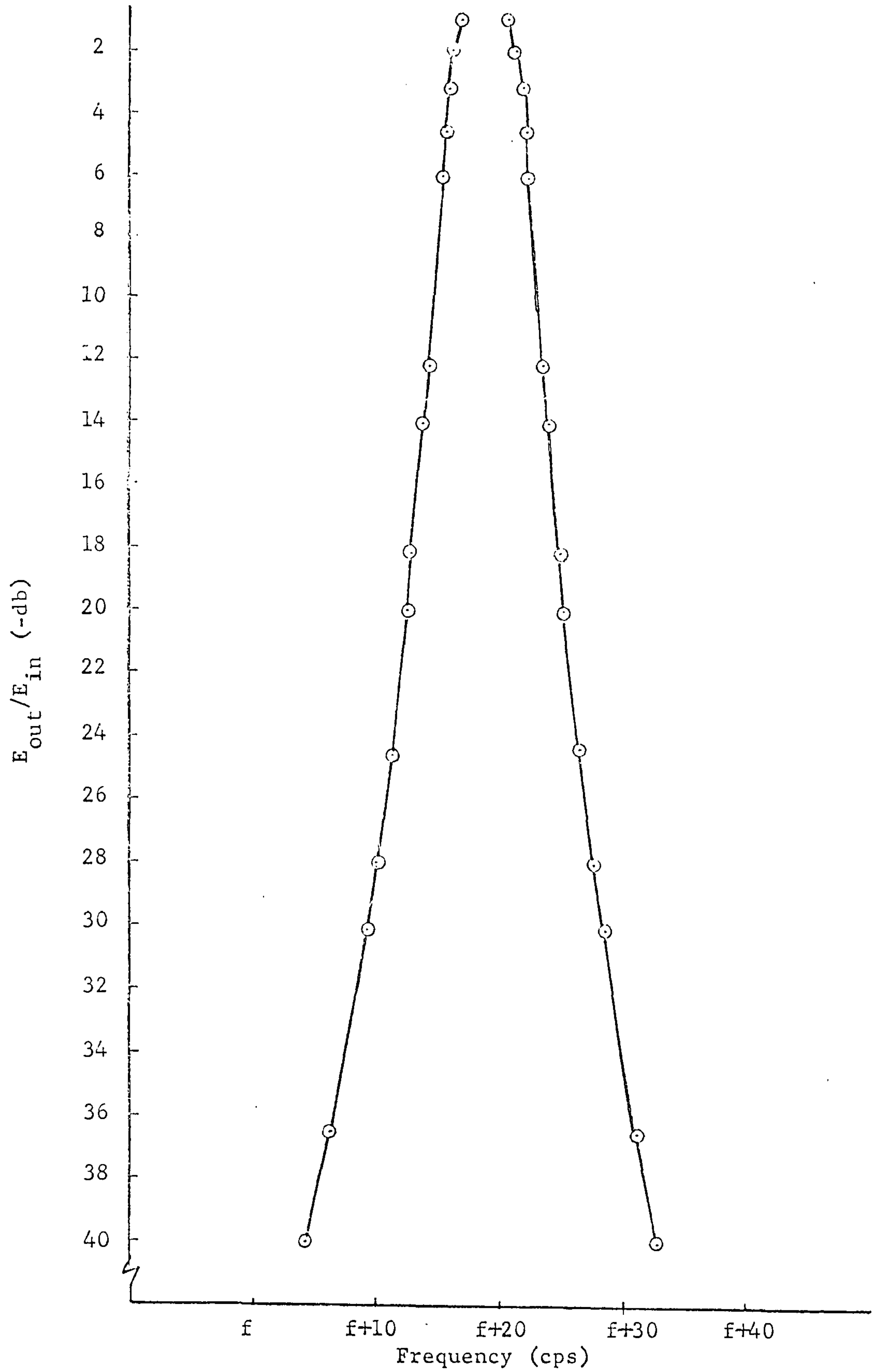

4
8
12
16
20
24
28
32
36
40
44
48
52
56
60
64
68
72
76
E_{out}/E_{in} (-db)
0
60
70
80
90
100
110
Frequency (cps)

Fig. 3. Measured E_{out}/E_{in} versus frequency for HP 302A wave analyzer



2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
E out/E in (-db)
f
f+10
f+20
f+30
f+40
Frequency (cps)

Fig. 4. Measured E_{out}/E_{in} versus frequency for the 440 cps tuning fork

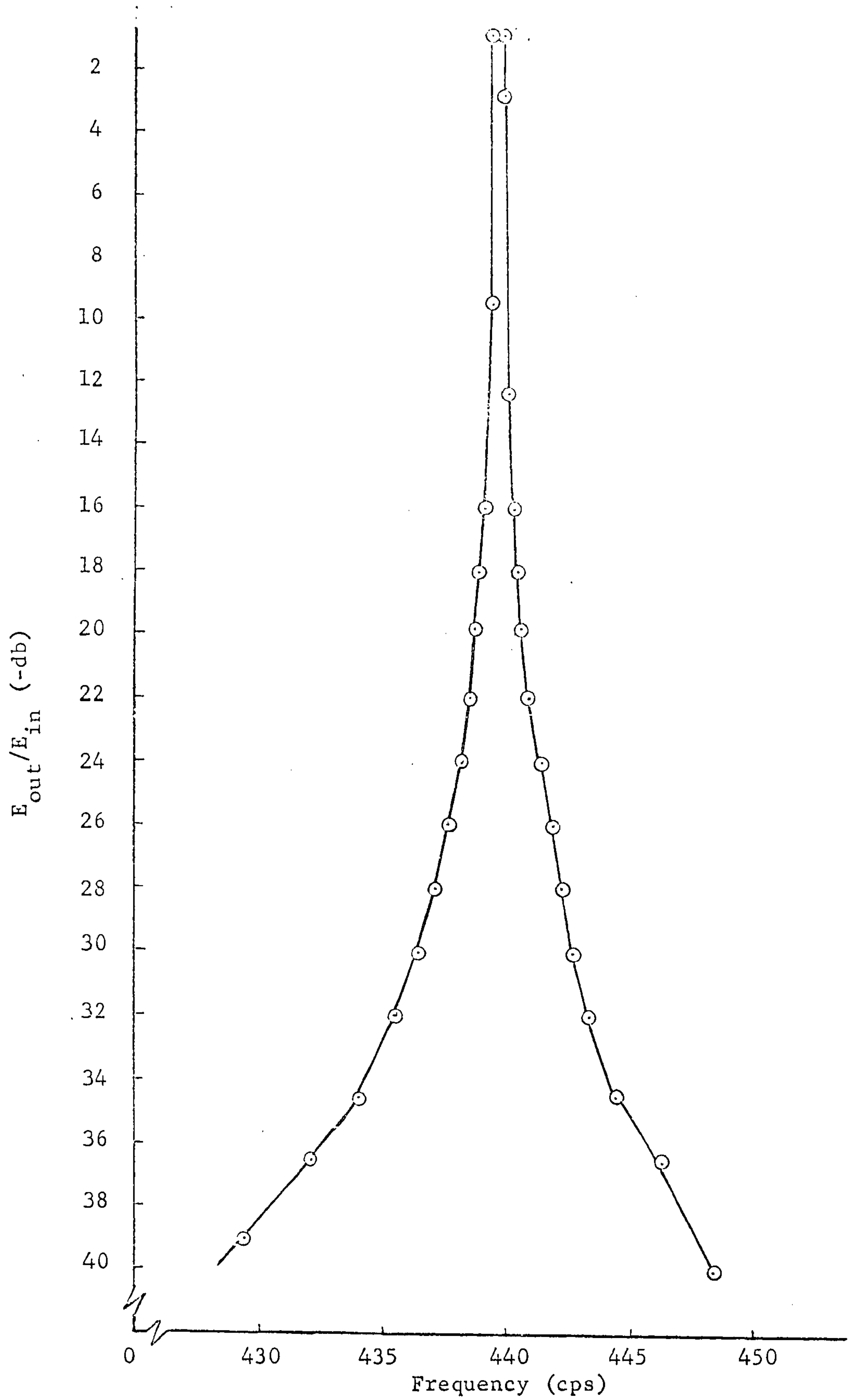

2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
E_{out}/E_{in} (-db)
0
430
435
440
445
450
Frequency (cps)

Fig. 5. The rms noise voltage versus frequency for a 120 volt 60 cps line

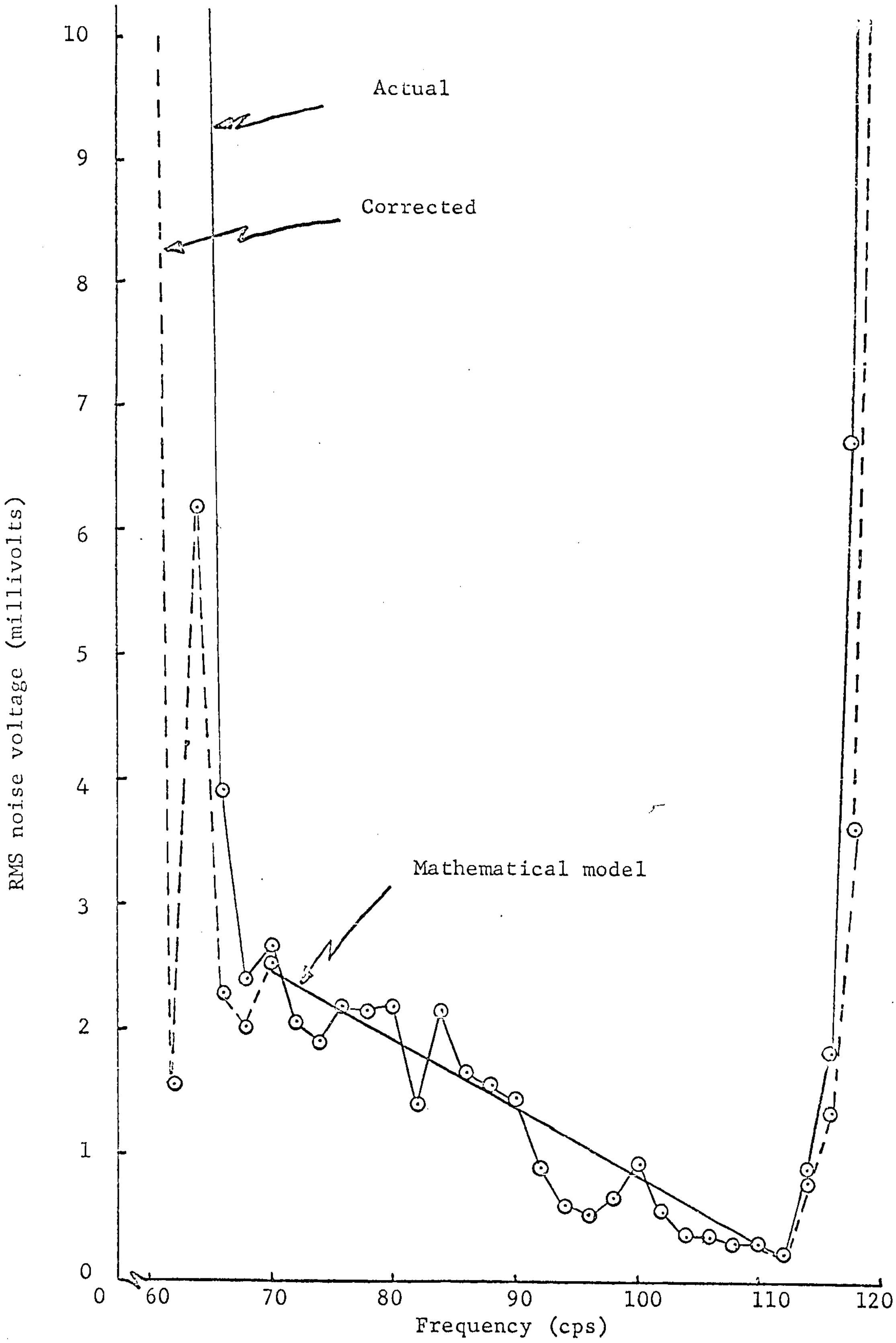
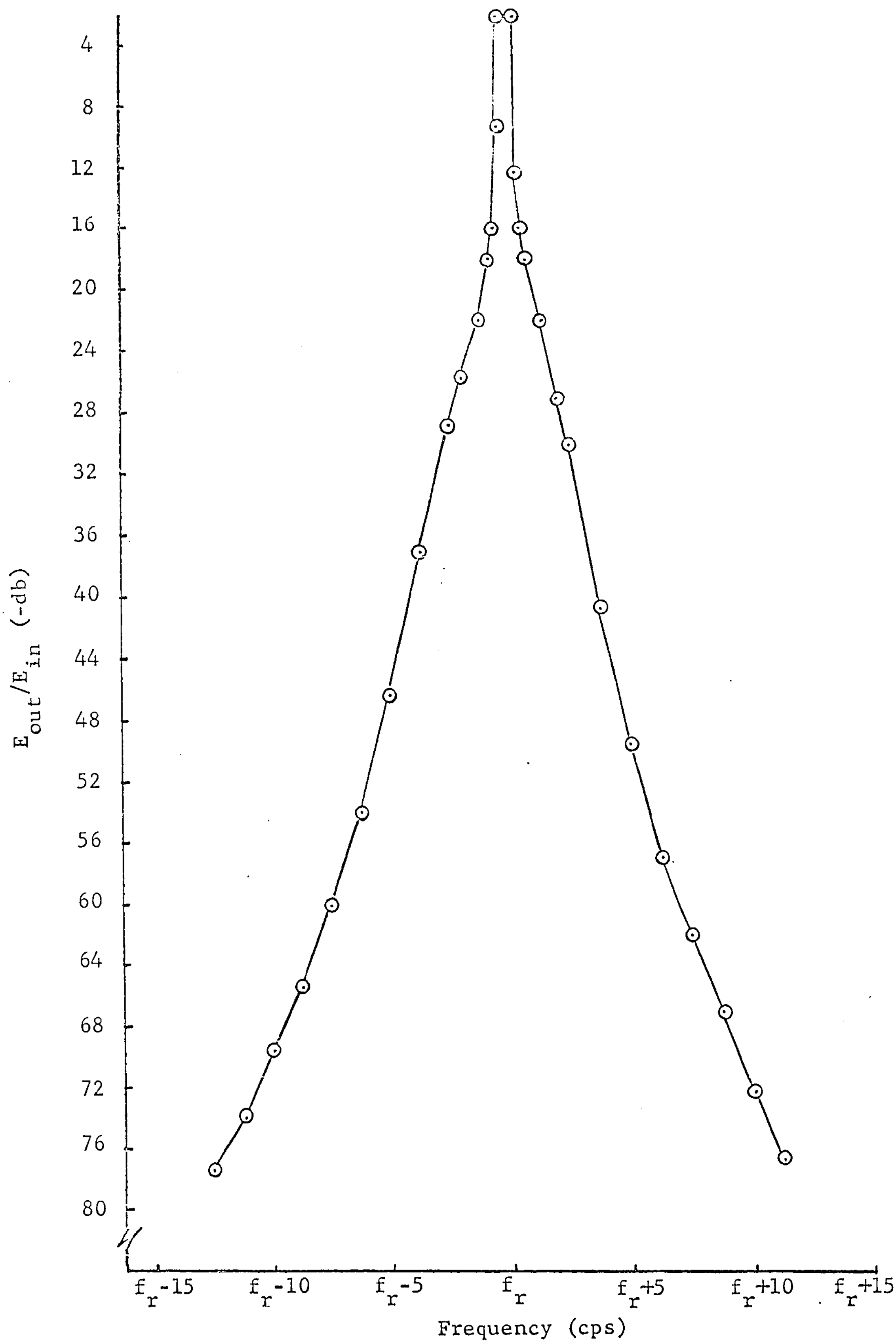

Actual
Corrected
Mathematical model
RMS noise voltage (millivolts)
Frequency (cps)
0
1
2
3
4
5
6
7
8
9
10
0
60
70
80
90
100
110
120

Table 1. Measured rms noise voltage versus frequency for a 120 volt 60 cps line

f cps	E_A mv	E_{out}/E_{in} -db	Mult.	V_L mv	f cps	E_A mv	E_{out}/E_{in} -db	Mult.	V_L mv
60	30.17	71.9	3980	120000	92	.186	13.8	4.90	.91
62	1.575	36.0	63.0	99.2	94	.136	13.4	4.68	.64
64	.567	30.0	33.0	18.7	96	.121	13.0	4.46	.54
66	.172	27.2	22.8	3.93	98	.161	12.6	4.27	.69
68	.146	24.4	16.6	2.43	100	.233	12.2	4.07	.95
70	.194	22.8	13.8	2.68	102	.144	11.8	3.89	.56
72	.175	21.4	11.8	2.09	104	.103	11.4	3.71	.38
74	.187	20.2	10.2	1.91	106	.109	11.0	3.55	.39
76	.243	19.2	9.1	2.21	108	.095	10.7	3.43	.32
78	.263	18.4	8.3	2.18	110	.098	10.4	3.31	.32
80	.290	17.6	7.6	2.20	112	.074	10.3	3.27	.24
82	.205	16.8	6.9	1.42	114	.285	10.1	3.20	.91
84	.334	16.2	6.45	2.16	116	.587	10.0	3.16	1.85
86	.279	15.6	6.02	1.68	118	2.077	9.9	3.13	6.50
88	.284	15.0	5.62	1.59	120	20.375	9.7	3.05	62.20
90	.280	14.4	5.25	1.47					

Fig. 6. Sum of HP 302A and tuning fork E_{out}/E_{in} versus frequency



4
8
12
16
20
24
28
32
36
40
44
48
52
56
60
64
68
72
76
80
E_{out}/E_{in} (-db)
f_r-15
f_r-10
f_r-5
f_r
f_r+5
f_r+10
f_r+15
Frequency (cps)

Table 2. Corrected rms noise voltage versus frequency for a 120 volt 60 cps line

f	Attenuation of 60 cps component	Attenuation of 120 cps component	Divider	Corrected line voltage
cps	db	db		mv
62	25.8		19.4	1.57
64	38.4		80.3	6.32
66	52.6		423	2.30
68	62.2		1290	2.04
70	69.6		3000	2.54
110		72.2	4030	.303
112		64.0	2500	.213
114		56.0	635	.807
116		42.8	138	1.38
118		27.0	22.4	3.65

that was present at the output of the twin-T filter was 30.17 mv as seen from Table 1. When the composite bandwidth characteristic was centered at 62 cps it was possible to read from Fig. 6 that the 60 cps component was down 25.8 db. The output voltage from the composite bandwidth was 1.55 mv for the 60 cps component. The measured voltage, E_A, at 62 cps, from Table 1, was 1.575 mv. The difference between these two readings gave a corrected reading of 0.025 mv. The corrected line voltage reading was the product of 0.025 and Mult. at 62 cps or 1.57 mv. The other four points around 60 cps were found in the same manner. To correct the reading around 120 cps it was necessary to assume an ideal line that gave 20.375 mv output from the twin-T filter and precede as above for the 60 cps component. Table 2 and Fig. 5 shows the values obtained for the corrected readings.

Current

The equipment shown in Fig. 7 was used to obtain the rms noise current variations on a 60 cps line over the frequency range from 66 to 114 cps. A current transformer was used to obtain a voltage proportional to the actual current in the line. The rms noise current measurements used the same procedure as the rms noise voltage measurements. The actual measurements of rms noise current per bandwidth versus frequency for a single conductor are shown in Figures 8 through 13 and Tables 3 through 8.

The measurements taken at the Electrical Engineering Building used a 40 to 1 current transformer with a secondary resistance of 0.05 ohms.

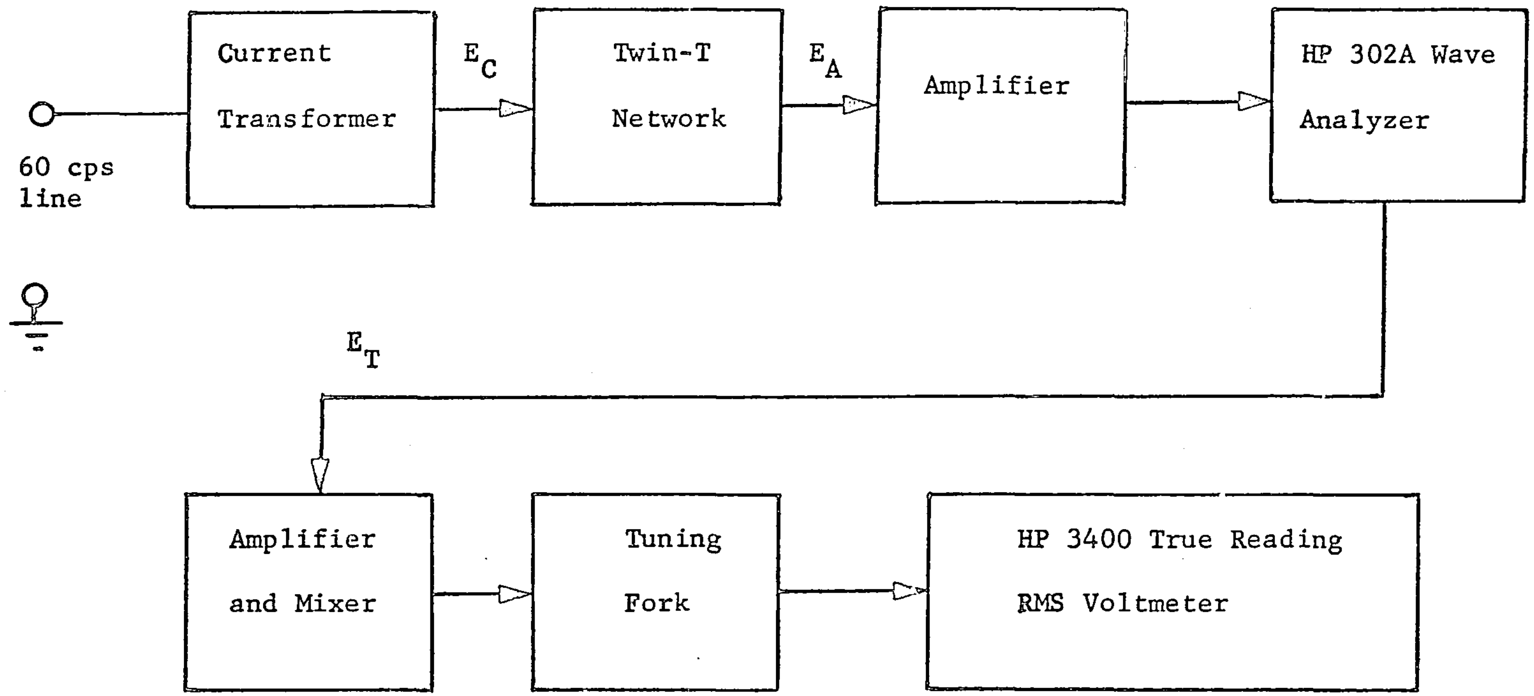


Fig. 7. Block diagram for measuring the rms noise current on a 60 cps line

The measurements taken at the Power Plant used a 40 to 1 current transformer with a secondary resistance of 0.647 ohms. In both readings the voltage E_C was given and it was the voltage across the secondary of the current transformer. To find the actual line current E_C was divided by the secondary resistance and then this quotient was multiplied by 40.

To compare the two sets of data taken at the Electrical Engineering Building and at the Power Plant it was necessary to have the same base current flowing in the conductor. The readings at the Electrical Engineering Building had average line currents of 97 and 106 amperes. The average of these two readings gave an average current of 101 amperes. The readings at the Power Plant had average line currents of 142 and 148 amperes. These two readings gave an average current of 145 amperes. It was known that rms currents in phase add directly. Using the definition of rms it was possible to see that currents of different frequencies add as the square root. It was assumed that the rms noise currents were completely random so that they added as the square root rather than linear.

The rms noise current for a conductor carrying 150 amperes was predicted. Multiplying the available data taken at the Electrical Engineering Building for a current of 101 amperes by the square root of 150/101 and the data taken at the Power Plant for a current of 145 amperes by the square root of 150/145 will give the predicted values for a conductor carrying an average current of 150 amperes (Fig. 14 and Table 9).

Mathematical Model for RMS Noise Versus Frequency

It is known that rms voltages or currents in phase add directly. Voltages or currents of different frequencies add as the square root. As an example, if two equal in phase voltages or currents are added the result is twice the magnitude. If two equal voltages or currents of different frequencies are added the resultant magnitude is increased by the square root of two. It has been assumed that the rms noise voltages or currents are completely random so they add as the square root rather than linear. It is now possible to predict the rms noise voltages or currents that are present for any line.

Voltage

Using the experimental results for rms noise voltage versus frequency obtained from Fig. 5 it is possible to fit a straight line that matches this curve in the 70 to 110 cps range reasonably well. The equation for this straight line is

$$V(f)_{120} = -(0.05357)(10^{-3})(f - 60) + (3)(10^{-3}) \tag{1}$$

Equation 1 applies when the average line voltage is 120 volts. The equation for the family of curves for any line voltage is

$$V(f)_{line} = (V_{line}/120)^{1/2}\, V(f)_{120} \tag{2}$$

Equation 2 can be used to find the rms noise voltage versus frequency for any line voltage, V_{line}, in the 70 to 110 cps range.

Current

Using the experimental results for rms noise current versus frequency obtained from Fig. 9 it is possible to fit a straight line that matches this curve in the 70 to 110 cps range reasonably well. The equation for this straight line is

$$I(f)_{106} = -(0.325)(10^{-3})(f - 60) + (23.2)(10^{-3}) \tag{3}$$

Equation 3 applies when the average line current is 106 amperes. The equation for the family of curves for any line current is

$$I(f)_{line} = (I_{line}/106)^{1/2}\ I(f)_{106} \tag{4}$$

Equation 4 can be used to find the rms noise current for any line current, I_{line}, in the 70 to 110 cps range. Equation 4 is plotted in Figures 8 through 13 for the proper I_{line}.

Fig. 8. Measured rms noise current versus frequency for a single conductor carrying an average current of 97 amperes located in the Electrical Engineering Building

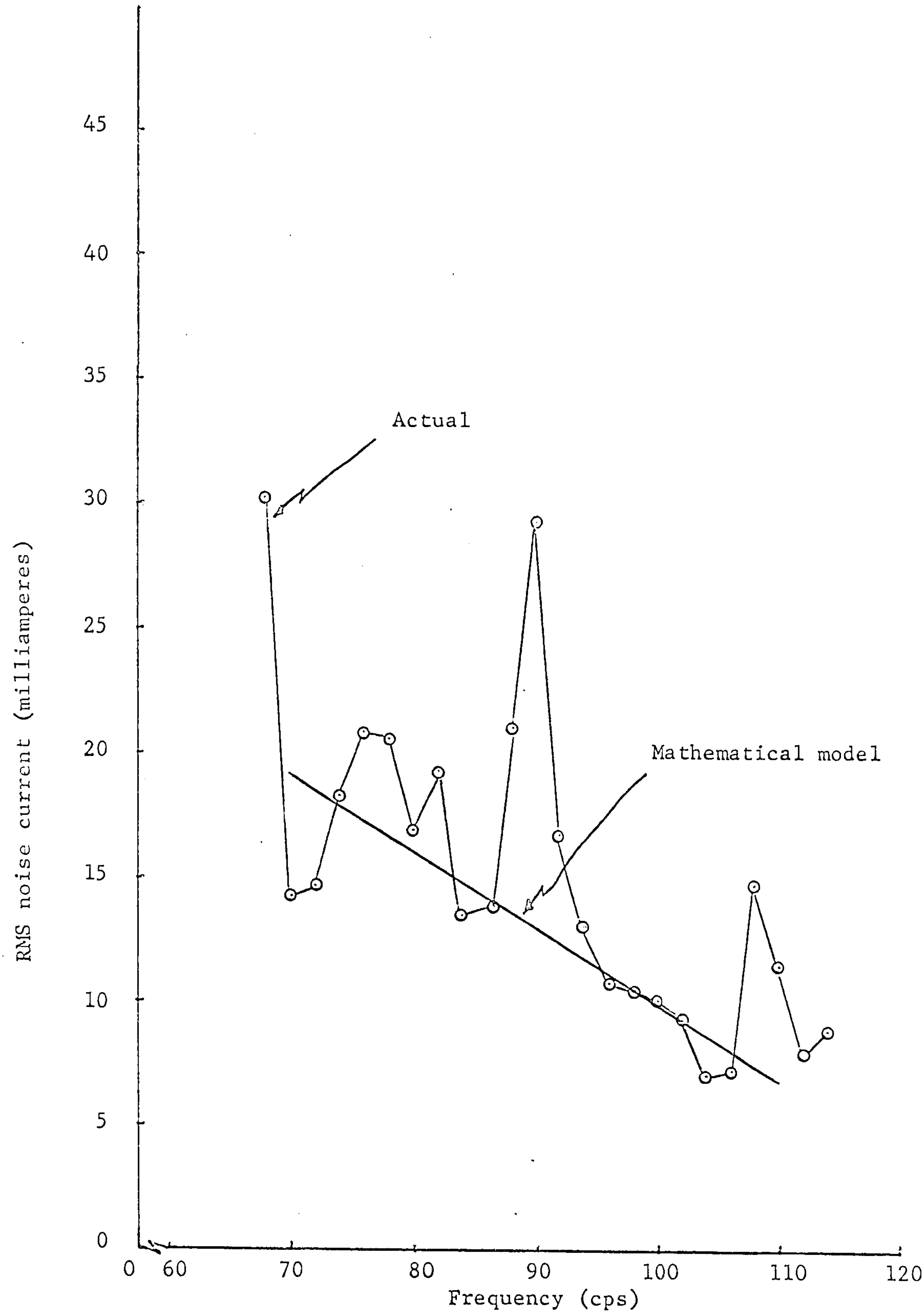
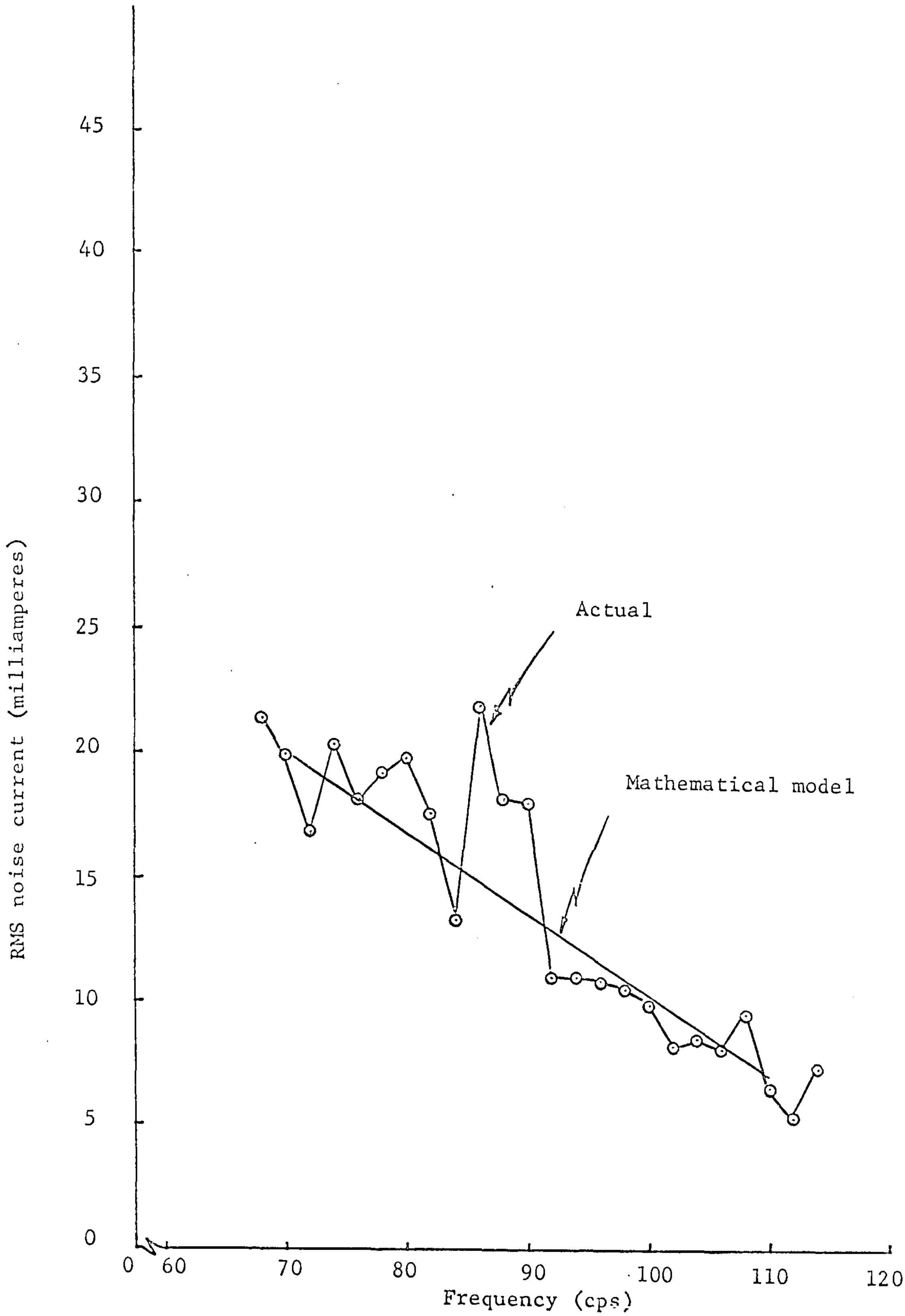

Actual
Mathematical model
RMS noise current (milliamperes)
0
5
10
15
20
25
30
35
40
45
0
60
70
80
90
100
110
120
Frequency (cps)

Table 3. Measured rms noise current versus frequency for a single conductor carrying an average current of 97 amperes located in the Electrical Engineering Building

f cps	E_A µv	Mult.	E_C µv	I_L ma	f cps	E_A µv	Mult.	E_C µv	I_L ma
66	4.26	22.8	97.1	77.6	92	4.31	4.90	21.10	16.85
68	2.27	16.6	37.7	30.2	94	3.50	4.68	16.37	13.08
70	1.29	13.8	17.82	14.26	96	3.02	4.46	13.45	10.78
72	1.57	11.8	18.45	14.75	98	3.08	4.27	13.13	10.51
74	2.24	10.2	22.85	18.30	100	3.11	4.07	12.65	10.12
76	2.87	9.1	26.10	20.90	102	3.02	3.89	11.72	9.38
78	3.11	8.3	25.82	20.62	104	2.37	3.71	8.79	7.04
80	2.80	7.6	21.25	17.00	106	2.55	3.55	9.05	7.24
82	3.49	6.9	24.10	19.28	108	5.42	3.43	18.60	14.88
84	2.61	6.45	16.82	13.45	110	4.31	3.31	14.25	11.41
86	2.89	6.02	17.42	13.92	112	3.02	3.27	9.90	7.92
88	4.68	5.62	26.25	21.00	114	3.48	3.20	11.10	8.88
90	6.98	5.25	36.60	29.30					

Fig. 9. Measured rms noise current versus frequency for a single conductor carrying an average current of 106 amperes located in the Electrical Engineering Building



RMS noise current (milliamperes)
0
5
10
15
20
25
30
35
40
45
Actual
Mathematical model
0
60
70
80
90
100
110
120
Frequency (cps)

Table 4. Measured rms noise current versus frequency for a single conductor carrying an average current of 106 amperes located in the Electrical Engineering Building

f cps	E_A μv	Mult.	E_C μv	I_L ma	f cps	E_A μv	Mult.	E_C μv	I_L ma
66	4.00	2.8	91.2	73.0	92	2.81	4.90	13.8	11.0
68	1.61	16.6	26.7	21.4	94	2.95	4.68	13.8	11.0
70	1.81	13.8	25.0	20.0	96	3.03	4.46	13.5	10.8
72	1.79	11.8	21.1	16.9	98	3.07	4.27	13.1	10.5
74	2.49	10.2	25.4	20.3	100	3.06	4.07	12.4	9.9
76	2.48	9.1	22.6	18.1	102	2.65	3.89	10.3	8.2
78	2.90	8.3	24.0	19.2	104	2.90	3.71	10.7	8.5
80	3.27	7.6	24.8	19.8	106	2.88	3.55	10.2	8.1
82	3.19	6.9	22.0	17.6	108	3.47	3.43	11.9	9.5
84	2.60	6.45	16.7	13.3	110	2.45	3.31	8.1	6.5
86	4.56	6.02	27.4	21.9	112	2.03	3.27	6.6	5.3
88	4.05	5.62	22.8	18.2	114	2.85	3.20	9.1	7.3
90	4.31	5.25	22.6	18.0					

Fig. 10. The average of the two rms noise current versus frequency readings taken at the Electrical Engineering Building. The average current was 101 amperes

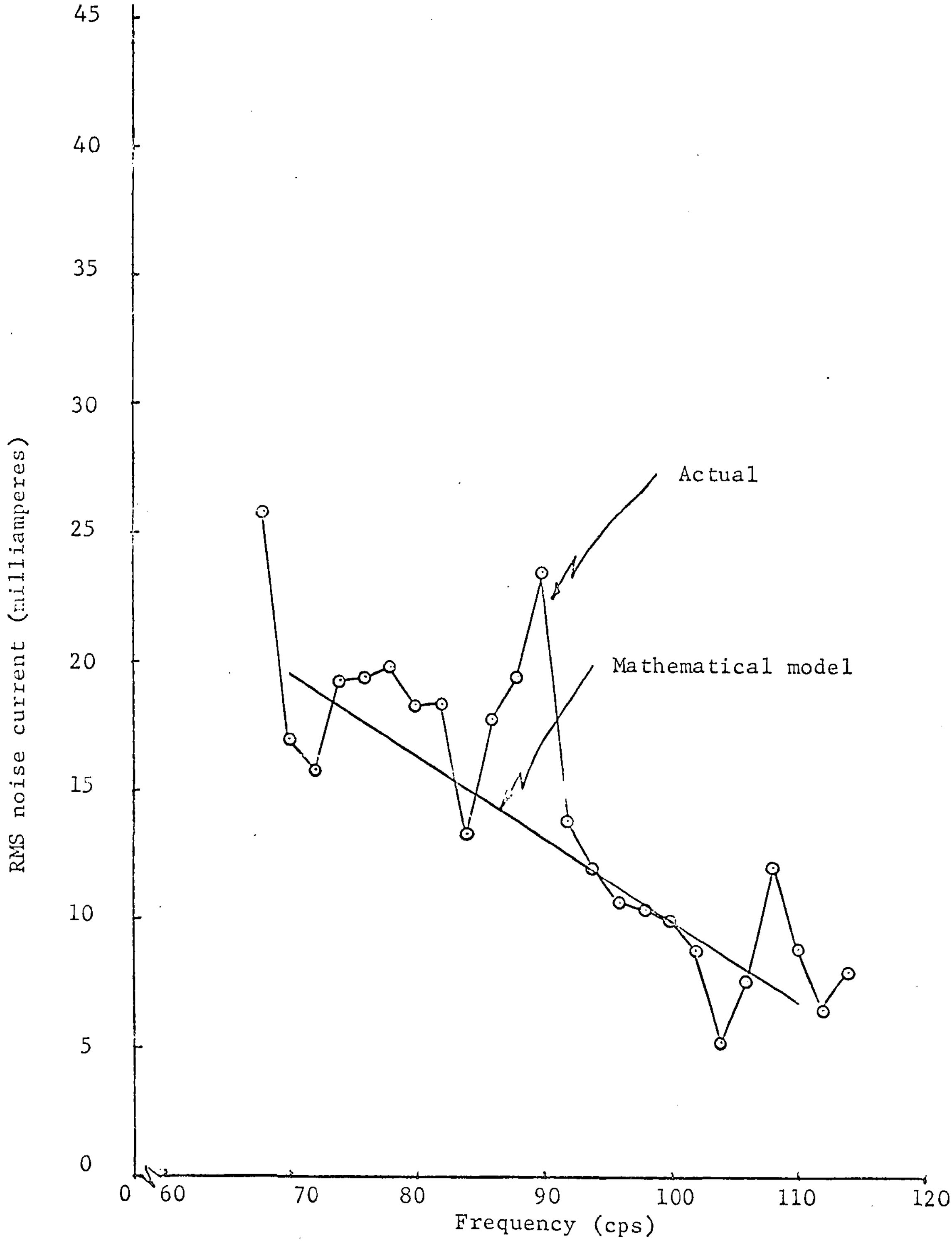

45
40
35
30
25
20
15
10
5
0
RMS noise current (milliamperes)
Actual
Mathematical model
0
60
70
80
90
100
110
120
Frequency (cps)

Table 5. The average of the two rms noise current versus frequency readings taken at the Electrical Engineering Building is 101 amperes

f cps	I_L ma	f cps	I_L ma
66	75.3	92	13.92
68	25.8	94	12.04
70	17.13	96	10.79
72	15.82	98	10.50
74	19.30	100	10.01
76	19.50	102	8.79
78	19.91	104	5.27
80	18.40	106	7.67
82	18.44	108	12.19
84	13.37	110	8.95
86	17.91	112	6.61
88	19.60	114	8.09
90	23.60		

Fig. 11. Measured rms noise current versus frequency for 60 cps line at the Power Plant with an average current of 142 amperes

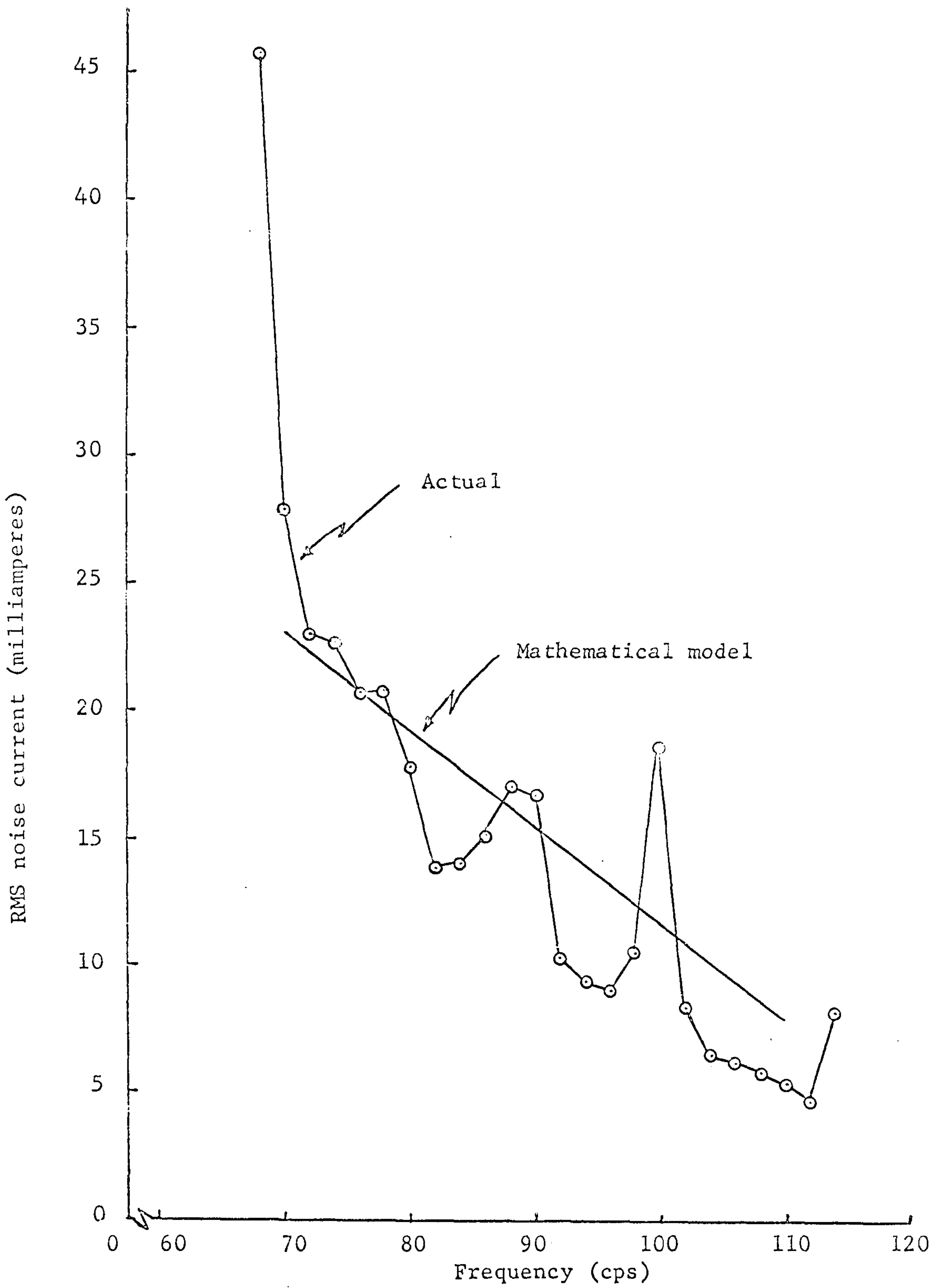

45
40
35
30
25
20
15
10
5
0
RMS noise current (milliamperes)
Actual
Mathematical model
0
60
70
80
90
100
110
120
Frequency (cps)

Table 6. Measured rms noise current versus frequency for 60 cps line at the Power Plant with an average current of 142 amperes

f cps	E_A μv	Mult.	E_C μv	I_L ma	f cps	E_A μv	Mult.	E_C μv	I_L ma
66	138.0	22.8	3142	195.0	92	34.3	4.90	168	10.40
68	44.6	16.6	741	45.8	94	32.7	4.68	153	9.46
70	32.8	13.8	452	27.9	96	33.0	4.46	147	9.10
72	31.7	11.8	374	23.1	98	44.1	4.27	188	10.63
74	36.0	10.2	367	22.7	100	74.4	4.07	303	18.75
76	36.9	9.1	336	20.8	102	35.1	3.89	136	8.42
78	40.7	8.3	338	20.9	104	28.7	3.71	107	6.62
80	38.2	7.6	290	17.95	106	28.4	3.55	101	6.25
82	32.4	6.9	224	13.85	108	28.1	3.43	96.4	5.96
84	35.3	6.45	228	14.10	110	26.6	3.31	88.0	5.44
86	40.6	6.02	246	15.22	112	23.4	3.27	76.6	4.74
88	49.3	5.62	277	17.15	114	41.4	3.20	133	8.23
90	52.0	5.25	273	16.90					

Fig. 12. Measured rms noise current versus frequency for 60 cps line at the Power Plant with an average current of 148 amperes

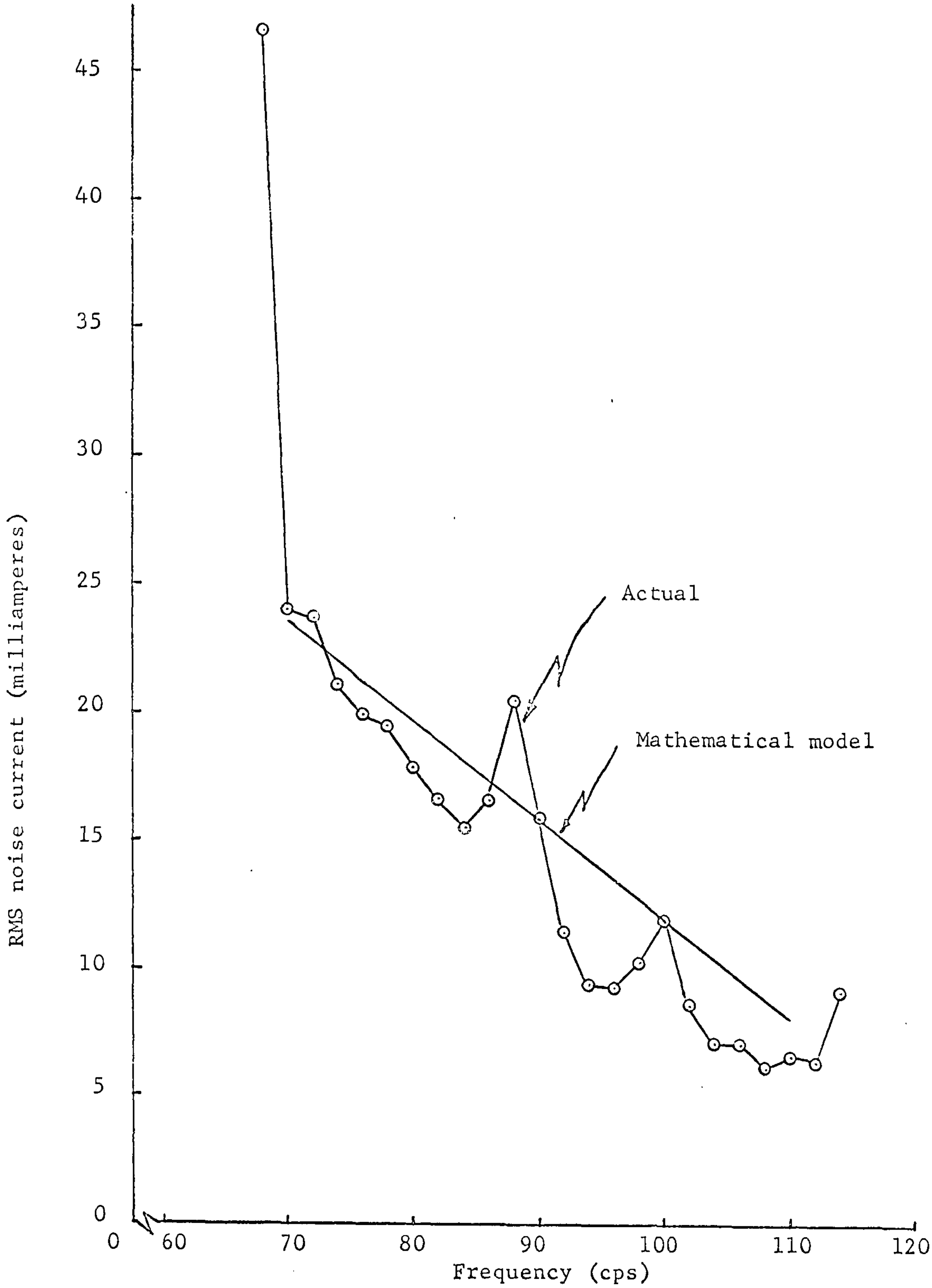
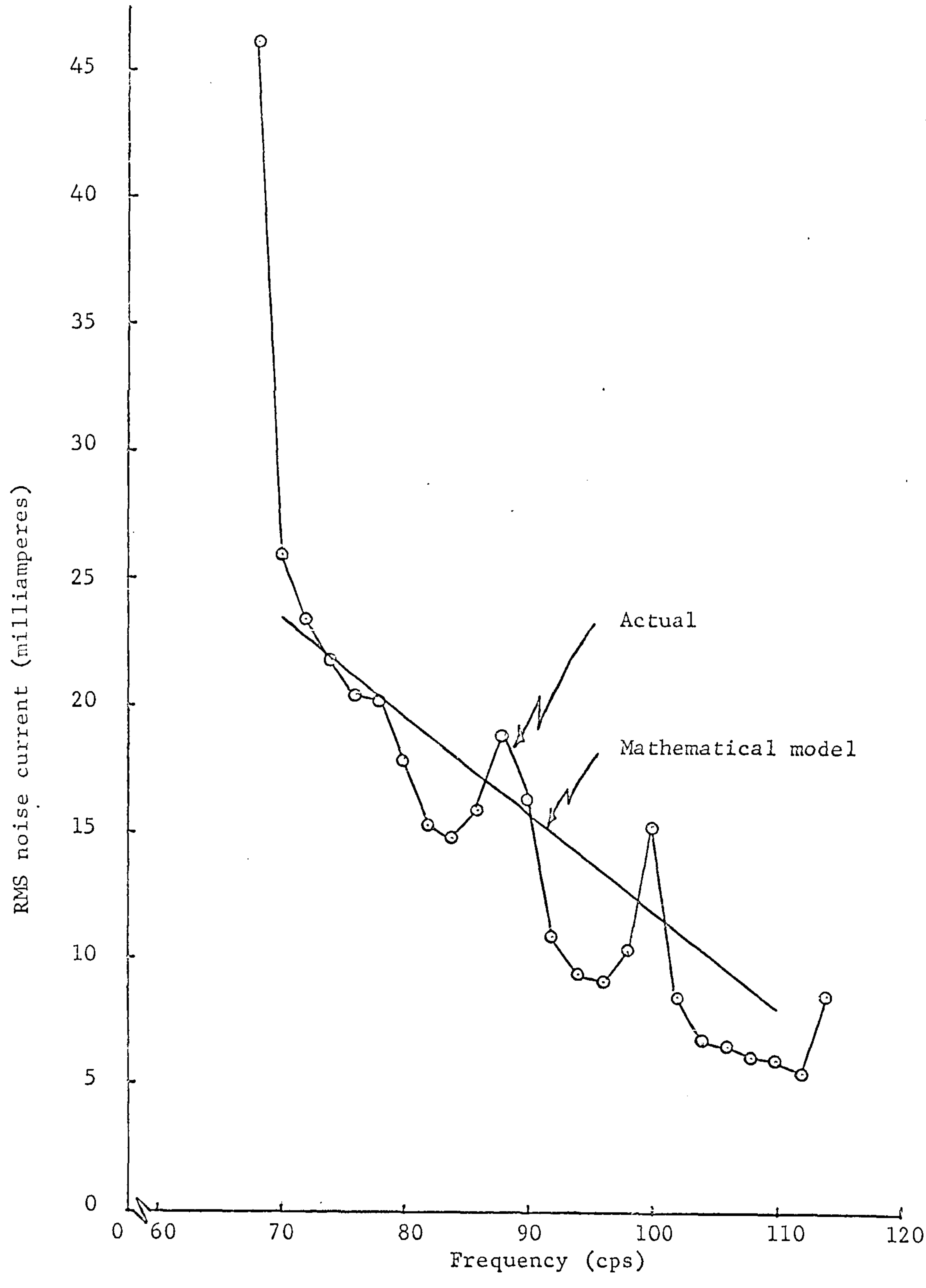

45
40
35
30
25
20
15
10
5
0
RMS noise current (milliamperes)
Actual
Mathematical model
0
60
70
80
90
100
110
120
Frequency (cps)

Table 7. Measured rms noise current versus frequency for 60 cps line at the Power Plant with an average current of 148 amperes

f cps	E_A μv	Mult.	E_C μv	I_L ma	f cps	E_A μv	Mult.	E_C μv	I_L ma
66	125	22.8	2850	173.3	92	37.7	4.90	185	11.45
68	45.3	16.6	753	46.6	94	32.3	4.68	152	9.41
70	28.1	13.8	388	24.0	96	33.6	4.46	150	9.28
72	32.4	11.8	383	23.7	98	39.0	4.27	166	10.28
74	33.4	10.2	341	21.1	100	47.2	4.07	192	11.88
76	35.5	9.1	323	20.0	102	35.9	3.89	139	8.60
78	38.1	8.3	316	19.55	104	31.1	3.71	115	7.12
80	38.1	7.6	290	17.95	106	32.2	3.55	114	7.05
82	39.1	6.9	270	16.70	108	29.1	3.43	100	6.19
84	39.0	6.45	252	15.60	110	32.7	3.31	108	6.68
86	44.8	6.02	270	16.70	112	31.6	3.27	103	6.38
88	59.0	5.62	331	20.50	114	46.0	3.20	147	9.10
90	49.2	5.25	258	15.95					

Fig. 13. The average of the two rms noise current versus frequency readings taken at the Power Plant. The average is 145 amperes



45
40
35
30
25
20
15
10
5
0
RMS noise current (milliamperes)
Actual
Mathematical model
0
60
70
80
90
100
110
120
Frequency (cps)

Table 8. The average of the two rms noise current versus frequency readings taken at the Power Plant is 145 amperes

f cps	I_L ma	f cps	I_L ma
66	185.6	92	10.92
68	46.2	94	9.43
70	25.9	96	9.19
72	23.4	98	10.45
74	21.9	100	15.31
76	20.4	102	8.51
78	20.22	104	6.87
80	17.95	106	6.65
82	15.27	108	6.07
84	14.85	110	6.06
86	15.96	112	5.51
88	18.82	114	8.66
90	16.42		

Fig. 14. Adjusted rms noise current versus frequency for both the average readings taken at the Electrical Engineering Building and the Power Plant. The adjusted readings are for a conductor carrying an average current of 150 amperes

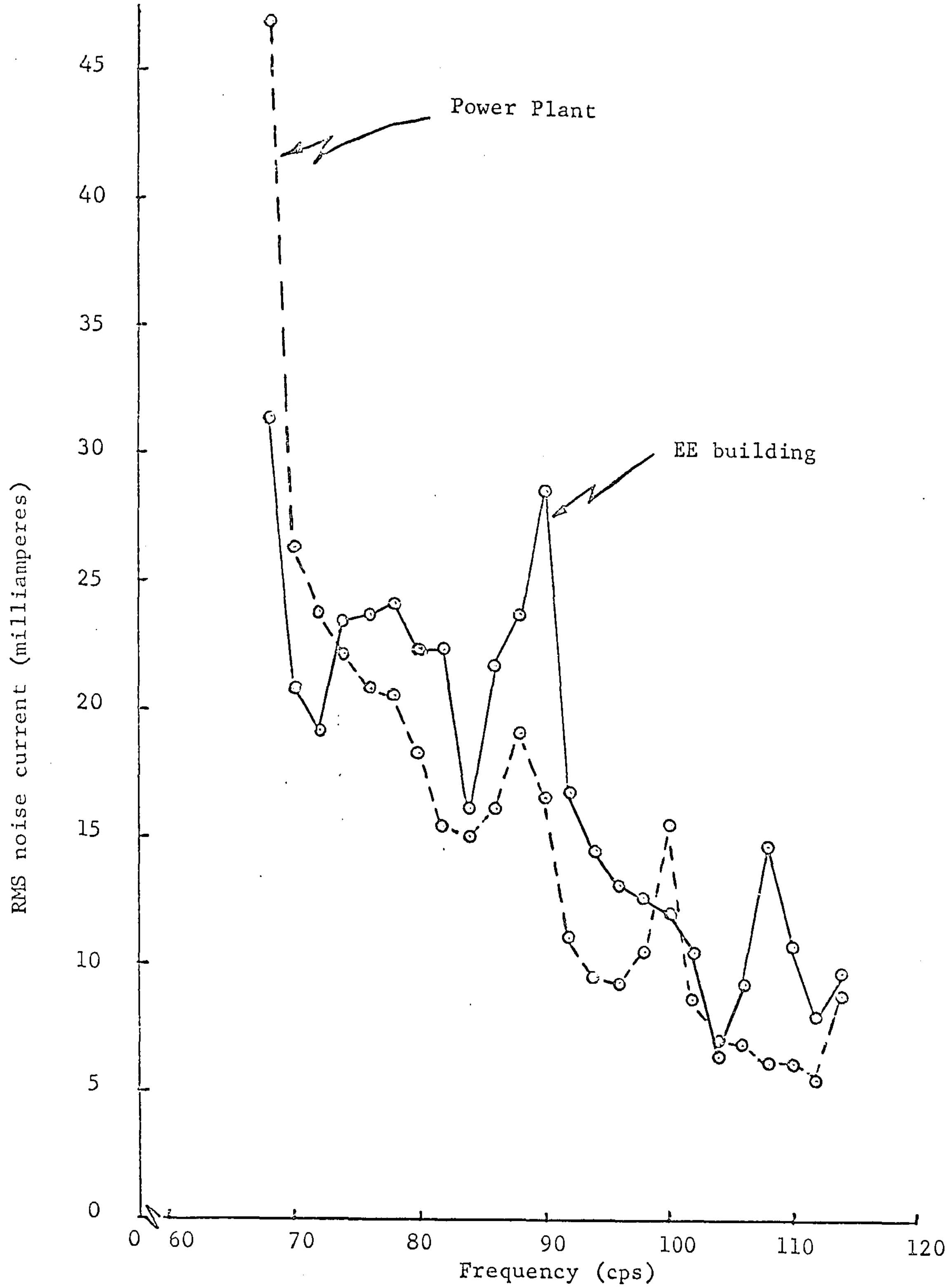

Power Plant
EE building
RMS noise current (milliamperes)
0
5
10
15
20
25
30
35
40
45
0
60
70
80
90
100
110
120
Frequency (cps)

Table 9. Adjusted rms noise current versus frequency for both the average readings taken at the Electrical Engineering Building and the Power Plant for a conductor carrying an average current of 150 amperes

f cps	I_{LEE} ma	I_{LPP} ma	f cps	I_{LEE} ma	I_{LPP} ma
66	91.6	188.5	92	16.9	11.1
68	31.4	47.0	94	14.6	9.6
70	20.8	26.3	96	13.2	9.3
72	19.2	23.8	98	12.7	10.6
74	23.5	22.2	100	12.1	15.6
76	23.7	20.8	102	10.6	8.7
78	24.2	20.5	104	6.4	7.0
80	22.4	18.3	106	9.3	6.8
82	22.4	15.5	108	14.8	6.2
84	16.2	15.1	110	10.9	6.2
86	21.8	16.2	112	8.1	5.6
88	23.8	19.1	114	9.8	8.8
90	28.6	16.7			

COMMUNICATING WITH THE SOURCE

Theory

Method of communication

There are at least three possible ways which a signal can be placed on a 60 cps transmission line in order to communicate with the source. The first method is to use a current transformer, power amplifier and a signal generator in the 70 to 110 cps range, to apply a fixed frequency to the line at any location and detect this signal at the source across resistor R. The second approach for communicating with the source utilizes the same equipment as the first method except the current is detected. The third method is to add a load to the line in a periodic manner so that certain frequency components in the current are present and detectable at the source.

The first method is treated in the simplified example shown in Fig. 15. It is assumed that r is very small and R is very large. The resistors R_1 and R_2 are the loads on the system. Using superposition to find the voltage V_1 due to e alone is

$$V_{1e} = \frac{e(rR_1/(r + R_1))}{(rR_1/(r + R_1))+R_2} = \frac{erR_1}{r(R_1+R_2)+R_1R_2} \tag{5}$$

$$\simeq er/R_2$$

The parallel combination of R and R_1 is R_1 because R is very large. The power supplied to R by e is

$$P_{Re} = V_{1e}^2/R \simeq (er/R_2)^2\ (1/R) \tag{6}$$
$$= (e^2/R)(r/R_2)^2$$

The noise power supplied to R by V in one-half cycle bandwidth is found from Fig. 5 to be

$$P_{NR} = e_N^2/R \tag{7}$$

To have a signal-plus-noise to noise-power ratio of 16 to 1 at R it follows that

$$(P_{Re} + P_{NR})/P_{NR} = 16 \tag{8}$$
$$P_{Re} = 15\ P_{NR}$$

Signal and noise power is used because both are referenced to the same value of resistance. The important quantity is the mean square value of the signal and of the noise. If equations 6 and 7 are substituted into equation 8 the result is

$$e = (15)^{\frac{1}{2}}\ e_N(R_2/r) \tag{9}$$

Equation 9 shows that the signal voltage e is a function of the load resistor R_2. It is possible for the resistor R_2 to become large enough so that e would have to be many volts to obtain the proper signal-to-noise ratio at R. The signal voltage e becomes a function of its physical position in the system which makes this method a poor choice.

The second method is also treated in the simplified example shown in Fig. 15. It is assumed that r is very small, R is very large, and R_1 and R_2 are the loads on the system. By placing a current transformer in the line between r and V_1 the current in the line is monitored by reading the voltage across the secondary resistance R_x. It is assumed that R_x is very small. Using superposition and the fact that R is very large the current I_e due to e alone is

$$I_e = \frac{e}{\frac{rR_1}{r+R_1} + R_2} \tag{10}$$

The parallel combination of r and R_1 is r because r is very small. The approximate value of I_e is

$$I_e \simeq e/R_2 \tag{11}$$

because r is much smaller than R_2. The value of the current i_e that flows through r due to e is

$$i_e = \frac{I_e R_1}{r+R_1} \simeq I_e \tag{12}$$

The power supplied to R_x by e is

$$P_{R_x e} = K^2 i_e^2 R_x \simeq K^2 I_e^2 R_x \tag{13}$$

where R_x is the resistor across the secondary of the monitoring current transformer and K is the turns ratio of the current transformer.

The noise power supplied to R_x by V in 1/2 cycle bandwidth is found from equation 4 to be

$$P_{NR_x} = K^2 i_N^2 R_x \tag{14}$$

To have a signal-plus-noise to noise-power ratio of 16 to 1 at R_x it follows that

$$(P_{R_x e} + P_{NR_x})/P_{NR_x} = 16$$

$$(P_{R_x e} = 15\ P_{NR_x}) \tag{15}$$

If equations 11, 12, and 14 are substituted into equation 15 the result is

$$e = (15)^{\frac{1}{2}}\ i_N R_2 \tag{16}$$

Equation 16 shows that the signal voltage e is a function of the load resistor R_2. The signal voltage e becomes a function of its physical position in the system which makes this second method a poor choice also.

The third method is treated in the simplified example shown in

Fig. 16. To communicate from any location back to the source it is necessary to place information on the line at a certain frequency and at a given rate so that it can be decoded at the source. It is possible to add a load to the line in a periodic manner so that certain frequency components will be present. The load must be added to and taken off the line at precise times in order to generate the proper frequency components in the signal that is sent to the source. If, when the voltages are exactly zero and going positive, loads are added to and taken off the line for exactly two complete cycles the frequency components in the signal are predictable. If a load is placed on the line for exactly two cycles and taken off for two cycles and repeated for a long time its frequency components can be found from a Fourier series.

It is seen in Fig. 16 that as the load R is switched on and off, the current i(t) is drawn from the source. By properly adjusting the switching time of R the current i(t) is made up of different frequency components. Once the maximum current I from the source is known it is possible to predict the value of R so that the frequency components of i(t) are larger than the noise components contained in a given bandwidth. The physical location of R is independent of the system, but its value depends on the system in which it is used. For communicating with the source this third method provides the best method to use. In the following section the Fourier series of i(t) is found.

Fourier series

If, i(t) as shown in Fig. 17 is the current that the periodic load

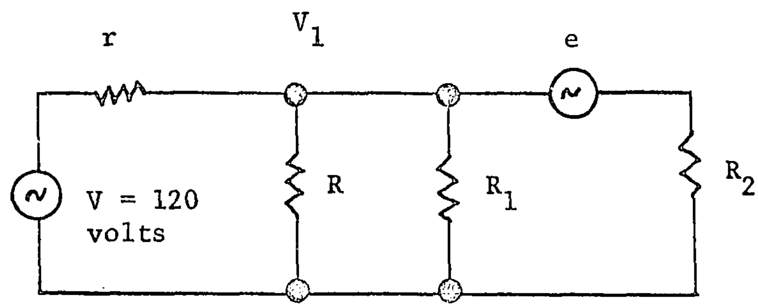


Fig. 15. Communicating with the source using a fixed frequency generator at any relative location

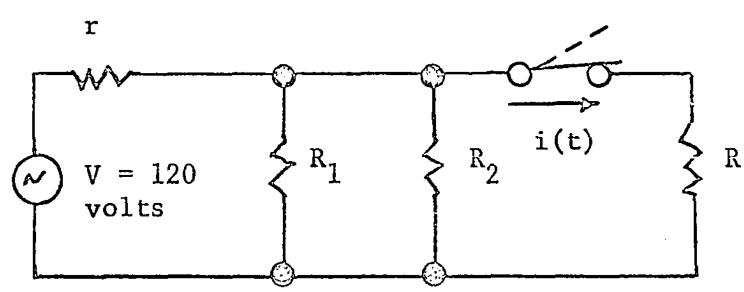


Fig. 16. Communicating with the source using a load R that is switched on and off the line in a predetermined manner

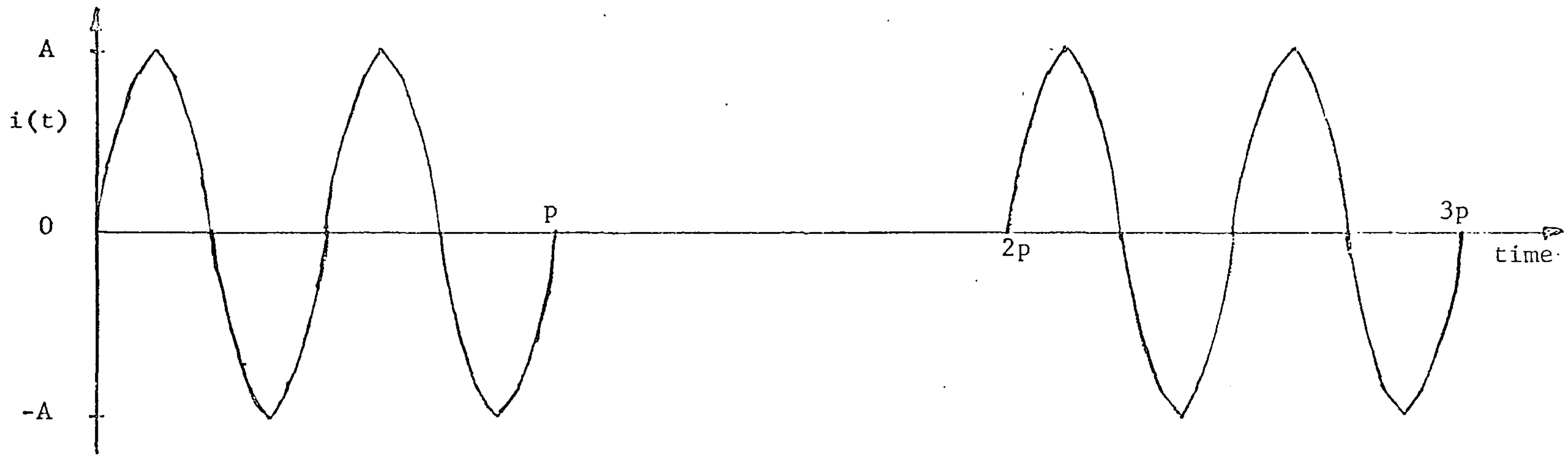


Fig. 17. The current i(t) is drawn by a periodic load that is placed on the line for p seconds and removed for p seconds

draws from the line, it is defined for all even m as

$$
\begin{aligned}
i(t) &= A \sin(\omega t) && mp \le t \le (m+1)p \\
&= 0 && (m+1)p \le t \le (m+2)p
\end{aligned}
\tag{17}
$$

The value of ω is $2\pi f = 120\pi$ radians per second. The period of $i(t)$ is $2p$ and its value is 1/15 seconds.

The Fourier series of $i(t)$ is (9)

$$
i(t) = \sum_{n=0}^{\infty} (a_n \cos(\pi n t/p) + b_n \sin(\pi n t/p)) \tag{18}
$$

where

$$
a_n = (1/p)\int_0^{2p} i(t) \cos(\pi n t/p)\, dt \tag{19}
$$

and

$$
b_n = (1/p)\int_0^{2p} i(t) \sin(\pi n t/p)\, dt \tag{20}
$$

The value of a_n for all n is found by performing the integration in equation 19 and using the following trigonometric identity

$$
\begin{aligned}
\sin(\omega t) \cos(\pi n t/p) &= (1/2)(\sin(\omega + \pi n/p)t + \sin(\omega - \pi n/p)t) \\
&= 1/2(\sin((4+n)\, 30\pi t) + \sin((4-n)\, 30\pi t))
\end{aligned}
\tag{21}
$$

Note that the second term in equation 21 is zero for $n = 4$ and the integral of the first term is zero between the limits of zero and $p = 1/30$. Therefore, $a_4 = 0$.

$$a_n = (A/2p) \int_0^p \left[\sin((4 + n)30\pi t) + \sin((4 - n)30\pi t) \right] dt \tag{22}$$

$$= (A/2\pi) \left[\frac{8}{16 - (n)(n)} - \frac{\cos((4 + n)\pi)}{4 + n} - \frac{\cos((4 - n)\pi)}{4 - n} \right]$$

The value of $\cos((4 + n)\pi)$ is equal to $\cos((4 - n)\pi)$ for all possible values of n.

Therefore,

$$a_n = (A/2\pi) \left[\frac{8}{16 - (n)(n)} - \frac{8 \cos((4 + n)\pi}{16 - (n)(n)} \right]$$

$$= 0 \quad \text{for n even} \tag{23}$$

$$\neq 0 \quad \text{for n odd}$$

The value of b_n for all n is found by performing the integration in equation 20 and using the following trigonometric identity

$$\sin(\omega t) \sin(\pi nt/p) = (1/2) \left[\cos((\omega - \pi n/p)t) - \cos((\omega + \pi n/p)t) \right] \tag{24}$$

$$= (1/2) \left[\cos((4 - n)30\pi t) - \cos((4 + n)30\pi t) \right]$$

Note that the first term in equation 24 is 1 for $n = 4$. The integral of $\cos(240\pi t)$ between the limits of zero and $p = 1/30$ is zero. Therefore,

$$b_4 = (A/2).$$

$$b_n = (A/2p) \int_0^p \left[\cos((4 - n)30\pi t) - \cos((4 + n)30\pi t)\right] dt$$

$$= (A/2\pi) \left[\frac{\sin((4 - n)\pi)}{4 - n} - \frac{\sin((4 + n)\pi)}{4 + n}\right] \tag{25}$$

$$= 0 \text{ for all } n \text{ except } n = 4.$$

The only values of a_n and b_n that are not zero are a_n for n odd and $b_4 = (A/2)$. The final Fourier series for i(t) given in equation 18 is the following

$$i(t) = (A/2)\sin(120\pi t) + \sum_{n(\text{odd})}^{\infty} (8A/\pi(16 - n^2))\cos(30\pi nt)$$

$$= (A/2)\sin((2\pi)60t) + (8A/15\pi)\cos((2\pi)15t) \tag{26}$$

$$+ (8A/7\pi)\cos((2\pi)45t) - (8A/9\pi)\cos((2\pi)75t)$$

$$- (8A/33\pi)\cos((2\pi)105t) - (8A/65\pi)\cos((2\pi)135t) - \ldots$$

The frequency components of the current waveform are 60, 15, 45, 75, 105, 135, cps, etc. These are the components present if a load is switched on and off the line in exactly two cycle intervals for a very long time. It is possible to communicate with the source at any of the distinct frequencies in the above series if the amplitude A is of sufficient magnitude. The magnitudes of the individual frequency components are equal at the load and at the source because the source provides the current used by the switched load.

Using a load that is switched on and off for a long period of time has limitations in a communication problem. The amount of information and the number of different locations that can send a signal to the source is limited. The number of sending locations could be increased by using a different switching time but the source would have to have a single receiver tuned to each frequency component of interest. Another solution to this problem is to decrease the time that each location is switched on and off.

Fourier integral

The problem of switching a load on and off the line for a short time is now investigated. The load is to be placed on the line for exactly two cycles and then removed for exactly two cycles and repeated for seven times as an example. The Fourier series of a function that is of finite length can not be found so the Fourier integral is used to find the magnitude of the time function in the frequency domain. The Fourier integral formulas which are called the Fourier transform are (10)

$$\underline{F}\left[i(t)\right] = F(\omega) = \int_{-\infty}^{\infty} i(t)\, e^{-j\omega t}\, dt \tag{27}$$

$$\underline{F}^{-1}\left[F(\omega)\right] = i(t) = \int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, df \tag{28}$$

To make the function i(t) as simple as possible and still derive the proper shape of the function in the frequency domain let i(t) be the product of two terms f(t) (Fig. 18) and $\sin(\omega_o t)$.

$$i(t) = f(t)\ \sin(\omega_o t)$$

$$= f(t)\ (e^{+j\omega_o t} - e^{-j\omega_o t})\ (1/2j) \tag{29}$$

The Fourier transform of i(t) is

$$F(\omega) = \int_{-\infty}^{\infty} i(t)\ e^{-j\omega t}\ dt$$

$$= \int_{-\infty}^{\infty} (f(t)/2j)\ e^{-j(\omega - \omega_o)t}\ dt - \int_{-\infty}^{\infty} (f(t)/2j)\ e^{-j(\omega + \omega_o)t}\ dt \tag{30}$$

$$= (1/2j)\ F(\omega - \omega_o) - (1/2j)\ F(\omega + \omega_o)$$

Equation 30 shows that i(t) is divided into two equal parts; $F(\omega - \omega_o)$ is centered at $\omega = \omega_o$ and $F(\omega + \omega_o)$ is centered at $\omega = -\omega_o$. If the Fourier transform of f(t) is known the Fourier transform of i(t) can be predicted by using equation 30.

Using equation 27 find the Fourier transform of f(t) as

$$\underline{F}\ \left[f(t)\right] = \int_{-\infty}^{\infty} f(t)\ e^{-j\omega t}\ dt$$

$$= \int_{-\infty}^{\infty} f(t)\ \cos(\omega t)\ dt - j \int_{-\infty}^{\infty} f(t)\ \sin(\omega t)\ dt \tag{31}$$

The second integral is zero because f(t) is an even function.

$$\underline{F}\left[f(t)\right] = 2\int_{o}^{13T/2} f(t)\cos(\omega t)\,dt$$

$$= (2A/\omega)\Big[\sin(\omega T/2) - \sin(\omega T3/2) + \sin(\omega T5/2) - \sin(\omega T7/2) + \sin(\omega T9/2) - \sin(\omega T11/2) + \sin(\omega T13/2)\Big] \qquad (32)$$

Equation 32 is written in more compact form as

$$\underline{F}\left[f(t)\right] = (2A/\omega)\sum_{n=0}^{6}\left[(-1)^n \sin((\omega)(2n+1)(T/2))\right] \qquad (33)$$

Equation 33 is for a function f(t) that has seven pulses in it. Note that n is summed over a total of seven numbers for this seven pulse function. Equation 33 could be used for any number of odd pulses by just changing the summation so long as the function f(t) is the same form as shown in Fig. 18.

If the Fourier transform of f(t) is plotted in the frequency domain it is an even function centered about f equal to zero. It is a continuous function but the envelope goes to zero with increasing frequency. There are peaks at $f = 0, \pm 15, \pm 45, \pm 75, \pm 105$ cps, etc. for $T = 1/30$ seconds. These peaks become very small for increasing values of frequency. The width of the peaks is changed by increasing or decreasing the number of pulses in f(t). By increasing the number of pulses in f(t) the width of the peaks become smaller and the amplitude of these peaks is increased. The reverse is true if the number

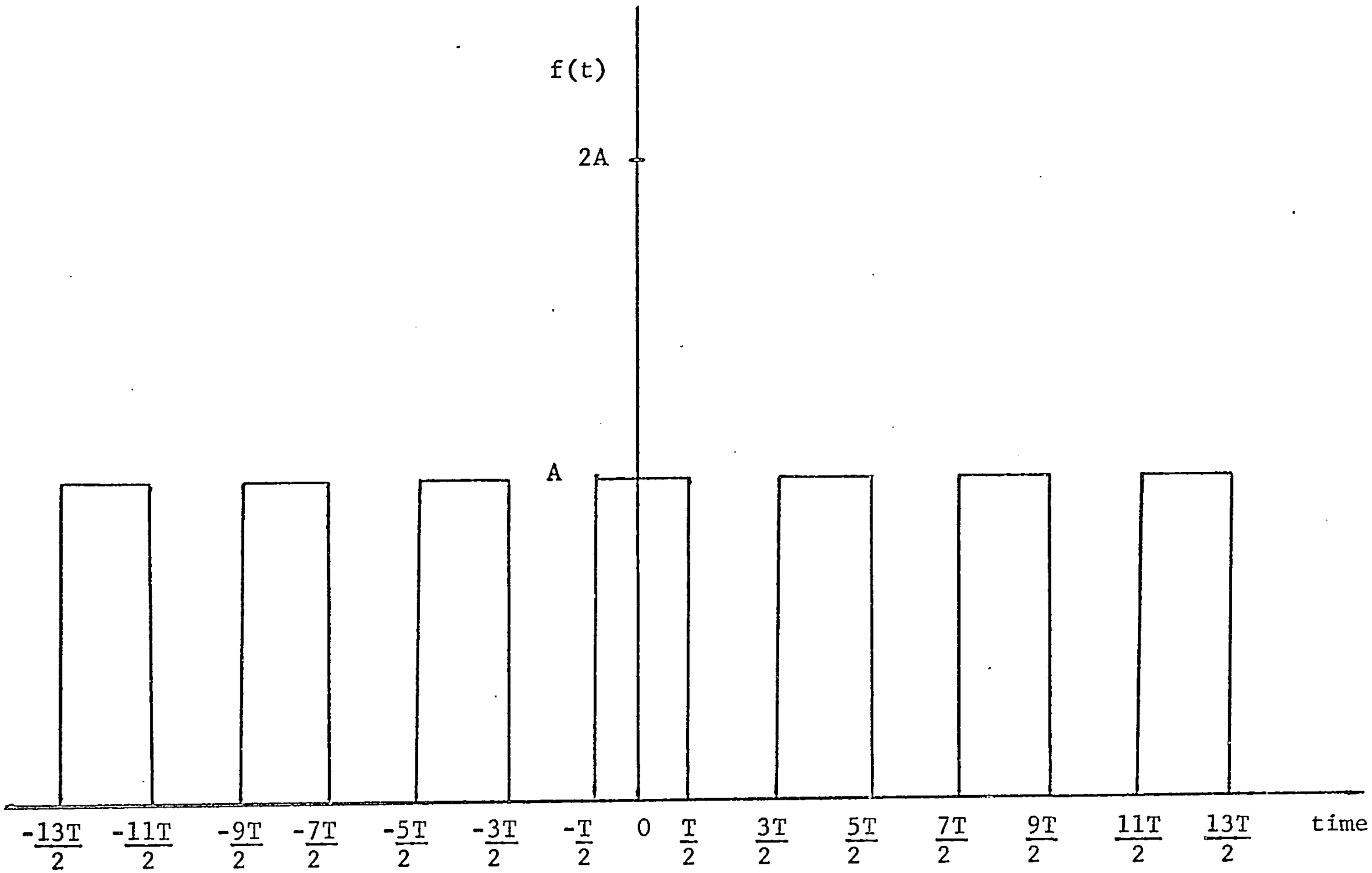


Fig. 18. The function f(t) is pulsed on and off every T seconds and is used in finding the Fourier transform of i(t)

of pulses in $f(t)$ is decreased.

Using equation 27 the Fourier transform of $i(t)$ is found and this shows that equation 30 is correct.

$$F(\omega) = \int_{-\infty}^{\infty} i(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-(13T/2)}^{(13T/2)} f(t)\, \sin(\omega_o t)\, \cos(\omega t)\, dt \tag{34}$$

$$-j \int_{-(13T/2)}^{(13T/2)} f(t)\, \sin(\omega_o t)\, \sin(\omega t)\, dt$$

The first integral in equation 34 is zero because $f(t)$ is even. The following trigonometric identity is used in equation 34

$$\sin(\omega_o t)\, \sin(\omega t) = (1/2)(\cos((\omega - \omega_o)t) - \cos((\omega + \omega_o)t)) \tag{35}$$

to obtain

$$F(\omega) = -2j \int_{0}^{(13T/2)} f(t)(1/2) \left[\cos((\omega - \omega_o)t) - \cos((\omega + \omega_o)t)\right] dt \tag{36}$$

After the integration is performed and written in closed form the result is

$$F(\omega) = (A/j(\omega - \omega_o)) \sum_{n=0}^{6} \left[(-1)^n \sin((\omega - \omega_o)(2n + 1)(T/2))\right]$$
$$- (A/j(\omega + \omega_o)) \sum_{n=0}^{6} \left[(-1)^n \sin((\omega + \omega_o)(2n + 1)(T/2))\right] \quad (37)$$

Comparing equations 33 and 37 it is possible to substantiate equation 30. Equation 37 is divided into two parts. First for positive frequencies it is centered at $\omega = \omega_o$ and second for negative frequencies it is centered at $\omega = -\omega_o$.

Equation 37 is the Fourier transform of $i(t) = f(t) \sin(\omega_o t)$. The function $f(t)$ is equal to A for T seconds then zero for T seconds and then it is repeated for seven times. The actual function $i(t)$ is equal to $A \sin(\omega_o t)$ for T seconds then zero for T seconds and this is repeated also for seven times. The function $i(t)$ is said to have seven current pulses in it. A general Fourier transform is written for any $i(t)$ that has an odd number, N, of these current pulses as

$$F(\omega)_N = (A/j(\omega - \omega_o)) \sum_{n=0}^{N} \left[(-1)^n \sin((\omega - \omega_o)(2n + 1)(T/2))\right]$$
$$- (A/j(\omega + \omega_o)) \sum_{n=0}^{N} \left[(-1)^n \sin((\omega + \omega_o)(2n + 1)(T/2))\right] \quad (38)$$

A general Fourier transform for an even number, M, of these current pulses in $i(t)$ is

$$F(\omega)_M = (A/j(\omega - \omega_o)) \sum_{n=1}^{M} \left[(-1)^n \sin((\omega - \omega_o)(2n - 1)(T/2))\right]$$
$$- (A/j(\omega + \omega_o)) \sum_{n=1}^{M} \left[(-1)^n \sin((\omega + \omega_o)(2n - 1)(T/2))\right] \tag{39}$$

Power spectral density

The power spectral density is defined as (11)

$$\Phi(\omega) = \lim_{T_1 \to \infty} (1/T_1) \left| F(\omega) \right|^2 \tag{40}$$

or

$$\Phi(\omega) = \int_{-\infty}^{\infty} R(\tau)\, e^{-j\omega\tau}\, d\tau \tag{41}$$

As before $F(\omega)$ is the Fourier transform of $i(t)$. The function $R(\tau)$ is called the autocorrelation function and is defined as

$$R(\tau) = \lim_{T_1 \to \infty} (1/T_1) \int_{-T_1/2}^{T_1/2} i(t)\, i(t + \tau)\, dt \tag{42}$$

Some of the properties of $R(\tau)$ are:

1. $R(0)$ is the mean square value of $i(t)$: $R(0) = \overline{i^2}$.

2. $R(\tau)$ is an even function $R(\tau) = R(-\tau)$.

3. The magnitude of $R(\tau)$ when $\tau = 0$ cannot be exceeded for any other value of τ: $R(0) \geq |R(\tau)|$.

A number of properties of $\Phi(\omega)$ can be deduced from its definition:

1. $\Phi(\omega)$ is an even function of ω.

2. $\Phi(\omega)$ is real.

3. The average power dissipated in a 1-ohm resistor by those frequency components of a current $i(t)$ lying in a band between f and $f + df$ is $2\Phi(\omega)\, df$.

4. $\Phi(\omega)$ is always positive.

From equations 40 and 42 it is seen that either $\Phi(\omega)$ or $R(\tau)$ is defined with the limit as T_1 approaches infinity. This implies that the power spectral density and the autocorrelation functions will define a time function for all time. Using the definitions for a time function of only finite length gives a value of zero for both the power spectral density and the autocorrelation.

The definitions of power spectral density and autocorrelation can be changed if the interpretation of the results are also changed. The power spectral density or autocorrelation of a finite time function is found if the limiting process in both definitions is removed and T_1 is thought of as the duration of the time function. This new power spectral density function is called a finite-time spectral density function and the new autocorrelation function is called a finite-time autocorrelation function. The finite-time spectral density or finite-time autocorrelation is now a true representation for only the time T_1. The finite-time spectral density and finite-time autocorrelation are defined for a time function of only limited duration as

$$\varphi(\omega) = (1/T_1) \left| F(\omega) \right|^2 \tag{43}$$

$$r(\tau) = (1/T_1) \int_{-T_1/2}^{T_1/2} i(t)\, i(t+\tau)\, dt \tag{44}$$

and

$$\varphi(\omega) = \underline{F} \left[r(\tau) \right] \tag{45}$$

The properties given before for power spectral density and autocorrelation are still valid under the new definitions.

To show that both definitions of $\varphi(\omega)$ give the same result for a general function $i_1(t)$ of finite time the following proof is given. The general function $i_1(t)$ is defined as

$$\begin{aligned} i_1(t) &= g(t) \quad -T/2 \leq t \leq T/2 \\ &= 0 \quad \left| t \right| > T/2 \end{aligned} \tag{46}$$

The time T for this example is finite.

The finite-time autocorrelation for g(t) is

$$r(\tau) = \frac{1}{T} \int_{-T/2}^{T/2} g(t)\, g(t+\tau)\, dt \qquad -T \leq \tau \leq T$$
$$= 0 \qquad |\tau| > T \tag{47}$$

Using equation 45, the finite-time spectral density for g(t) is

$$\varphi(\omega) = \underline{F}\left[r(\tau)\right]$$
$$= \int_{-\infty}^{\infty} r(\tau)\, e^{-j\omega\tau}\, d\tau$$
$$= \int_{-T}^{T} r(\tau)\, e^{-j\omega\tau}\, d\tau \tag{48}$$

Equation 47 is used to fix the limits on the integral in equation 48. Equation 47 is substituted in equation 48 and the result is

$$\varphi(\omega) = \int_{-T}^{T} \frac{1}{T} \int_{-T/2}^{T/2} g(v)\, g(v+\tau)\, dv\, e^{-j\omega\tau}\, d\tau \tag{49}$$

The order of integration is interchanged so

$$\varphi(\omega) = \frac{1}{T} \int_{-T/2}^{T/2} \int_{-T}^{T} g(v)\, g(v+\tau)\, e^{-j\omega\tau}\, d\tau dv \tag{50}$$

The variable $(v + \tau)$ is replaced by u in the first integral. The variable v in the integral is a constant and arbitrary.

$$\varphi(\omega) = \frac{1}{T} \int_{-T/2}^{T/2} \int_{-T+v}^{T+v} g(v)\, g(u)\, e^{-j\omega(u - v)}\, dudv$$

$$= \frac{1}{T} \int_{-\frac{T}{2}}^{T/2} g(v)\, e^{+j\omega v} \int_{-T+v}^{T+v} g(u)\, e^{-j\omega u}\, dudv \tag{51}$$

The maximum value that v has is T/2 because g(v) for v greater than T/2 is zero. As v is constant in the first integral with respect to u set u equal to a value less than T/2. After doing this break the first integral into three parts.

$$\varphi(\omega) = \frac{1}{T} \int_{-T/2}^{T/2} g(v)\, e^{+j\omega v} \left[\int_{-T+v}^{-T/2} g(u)\, e^{-j\omega u} \right.$$

$$\left. + \int_{-T/2}^{T/2} g(u)\, e^{-j\omega u} + \int_{T/2}^{T+v} g(u)\, e^{-j\omega u} \right] dudv \tag{52}$$

The first and third terms in equation 52 are zero because the function g(u) is zero when the magnitude of u is greater than T/2. Equation 52 reduces to

$$\varphi(\omega) = \frac{1}{T} \int_{-T/2}^{T/2} g(v)\ e^{+j\omega v}\ dv \int_{-T/2}^{T/2} g(u)\ e^{-j\omega u}\ du \tag{53}$$

Equation 53 is the same as

$$\begin{aligned} \varphi(\omega) &= \frac{1}{T}\ F(-\omega)\ F(\omega) \\ &= \frac{1}{T}\ \left| F\ (\omega) \right|^2 \end{aligned} \tag{54}$$

It was possible to start with equation 48 and develop equation 54. These two equations are the same as equations 45 and 43 respectively. This proof shows that $\varphi(\omega)$ as redefined is useable for a finite time function. Equation 43 is used in the rest of this paper to find $\varphi(\omega)$ for finite time functions.

Power spectral density is used because the integral of $\Phi(\omega)$ is equal to the total power supplied (12). From the definition of power spectral density, equation 40, it is seen that the total power is

$$\text{Power} = \int_{-\infty}^{\infty} \Phi(\omega)\ df \tag{55}$$

The power spectral density $\Phi(\omega)$ is a real, symmetrical function. The power in the frequency range f_1 to f_2 is given by

$$\int_{f_1}^{f_2} 2\Phi(\omega)\, df \tag{56}$$

The factor 2 occurs here because in the definition of power spectral density one-half the power is assigned to negative frequencies. This general definition is independent of time and holds for all time.

The same result can be obtained for a time function of limited duration, but the results are only good during a finite time period. For a time function of limited duration the total average power is

$$\text{Average Power} = \int_{-\infty}^{\infty} \varphi(\omega)\, df \tag{57}$$

The average power in the frequency range f_1 to f_2 is given by

$$\int_{f_1}^{f_2} 2\varphi(\omega)\, df \tag{58}$$

Equations 57 and 58 are only good for the period T_1, which corresponds to the time that i(t) has a value.

It is necessary to go to information theory to determine the theoretical minimum length of time that a binary bit of information must be present in order to communicate from one location to another. The channel capacity, C, in bits is (13)

$$C = W \log_2 (1 + P/N) \tag{59}$$

W is the bandwidth of the receiver. P is the signal power and N is the noise power. Equation 59 signifies that by sufficiently involved encoding it is possible to transmit binary information at the rate C per second, with an arbitrarily small frequency of errors. C is also the theoretical upper limit for the rate of transmission. For a bandwidth of about one-half cycle the rate is about one to two bits per second. This implies that the theoretical minimum length of time that an information bit must be present is one-half to one second.

Mathematical Model

Computer program

A computer program was written to evaluate the finite-time spectral density of a 7, 11, and 15 pulse information bit. Each pulse was on for T = 1/30 seconds and then off for T = 1/30 seconds and this was repeated 7, 11, and 15 times respectively. Equation 38 gives the Fourier transform of $i_N(t)$ for any odd N. The finite-time spectral density of $i_N(t)$ for any odd N is

$$\varphi(\omega)_N = (A^2/T_{1N}(\omega - \omega_o)^2) \left[\sum_{n=0}^{N-1} ((-1)^n \sin((\omega - \omega_o)(2n+1)(T/2))) \right]^2$$

$$+ (A^2/T_{1N}(\omega + \omega_o)^2) \left[\sum_{n=0}^{N-1} ((-1)^n \sin((\omega + \omega_o)(2n+1)(T/2))) \right]^2$$

$$- \left[(2A^2 / T_{1N}(\omega - \omega_o)\ (\omega + \omega_o)) \right]$$

$$\left[\sum_{n=0}^{N-1} ((-1)^n \sin((\omega - \omega_o)(2n + 1)T/2)) \right] \tag{60}$$

$$\left[\sum_{n=0}^{N-1} ((-1)^n \sin((\omega + \omega_o)\ (2n + 1)T/2)) \right]$$

Using $\omega_o = 2\pi(60)$ radians per second and $T = 1/30$ seconds equation 60 becomes

$$\varphi(\omega)_N = \left[\sum_{n=0}^{N-1} ((-1)^n \sin((\omega)(2n + 1)(T/2))) \right]^2$$

$$(A^2/T_{1N}) \left[(1/(\omega - \omega_o)^2) + (1/(\omega + \omega_o)^2) - (2/(\omega - \omega_o)(\omega + \omega_o)) \right]$$

$$\varphi(\omega)_N = \frac{A^2 \ 4\ \omega_o^{\ 2}}{T_{1N}(\omega^2 - \omega_o^{\ 2})^2} \left[\sum_{n=0}^{N-1} ((-1)^n \sin((\omega)(2n + 1)T/2)) \right]^2 \tag{61}$$

Equation 61 is the finite-time spectral density function for $i_N(t)$ of the form given in equation 29 for any odd number of pulses. The value of $T_{1N} = (2N - 1)T$ for all odd N. The value of A is one for this program. The finite-time spectral density of a seven pulse information bit implies that N = 7.

A Fortran computer program was written that evaluates equation 61 for $\varphi(\omega)_7$, $\varphi(\omega)_{11}$, and $\varphi(\omega)_{15}$. These values are the finite-time spectral densities of $i_7(t)$, $i_{11}(t)$, and $i_{15}(t)$ respectively. The subscript implies the number of current pulses in each information bit. The frequency of interest is around 75 cps because $\varphi(\omega)_N$ has its first peak at 75 cps and this is in the range of the actual measurements taken. Points for $\varphi(\omega)_N$ are obtained from 74 to 76 cps (Fig. 19).

The actual power contained in $\varphi(\omega)_N$ is also of interest. A plot of the actual power versus bandwidth for $\varphi(\omega)_N$ centered about 75 cps is shown in Fig. 20. It is noted that the power is the integral of $2\varphi(\omega)_N df$ because one-half the power is assigned to negative frequencies.

Detecting and Amount of Current Signal

Method

A current signal that is large enough to be detected is placed on the line by adding a load at a remote location. This load is placed on the line for exactly two complete cycles and taken off for two complete cycles and repeated for seven times. The finite-time spectral density of this current signal has a peak at 75 cps that is detectable. To detect this current signal at 75 cps a current transformer is placed around the conductor and a narrow band receiver is used.

If the equipment shown in Fig. 7 is used the HP 302A is tuned to 75 cps and the mixer circuit is omitted and a 150 cps tuning fork is used.

Fig. 19. Plots of finite-time spectral density for $\varphi(\omega)_7$, $\varphi(\omega)_{11}$, and $\varphi(\omega)_{15}$ versus frequency around 75 cps

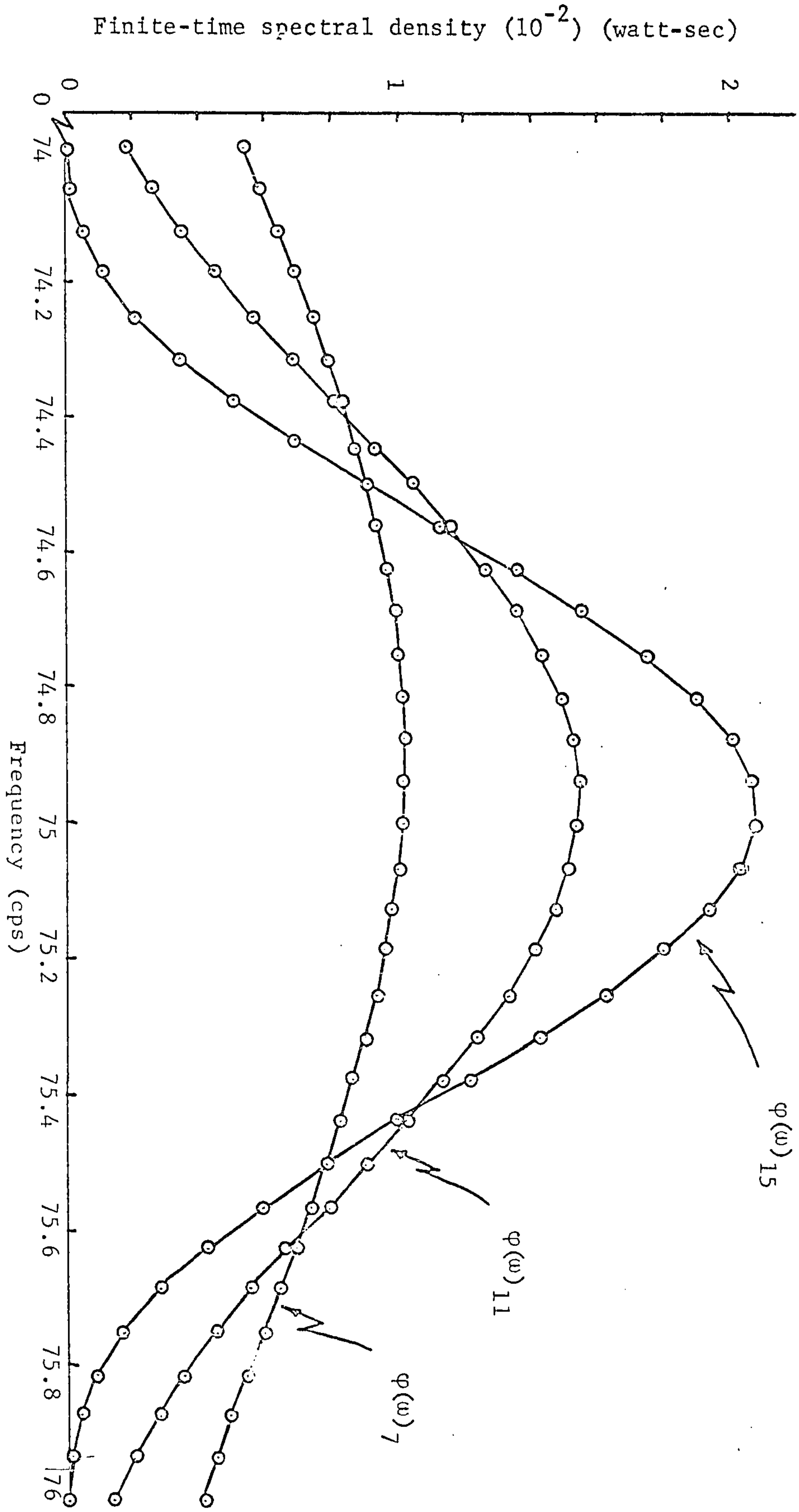
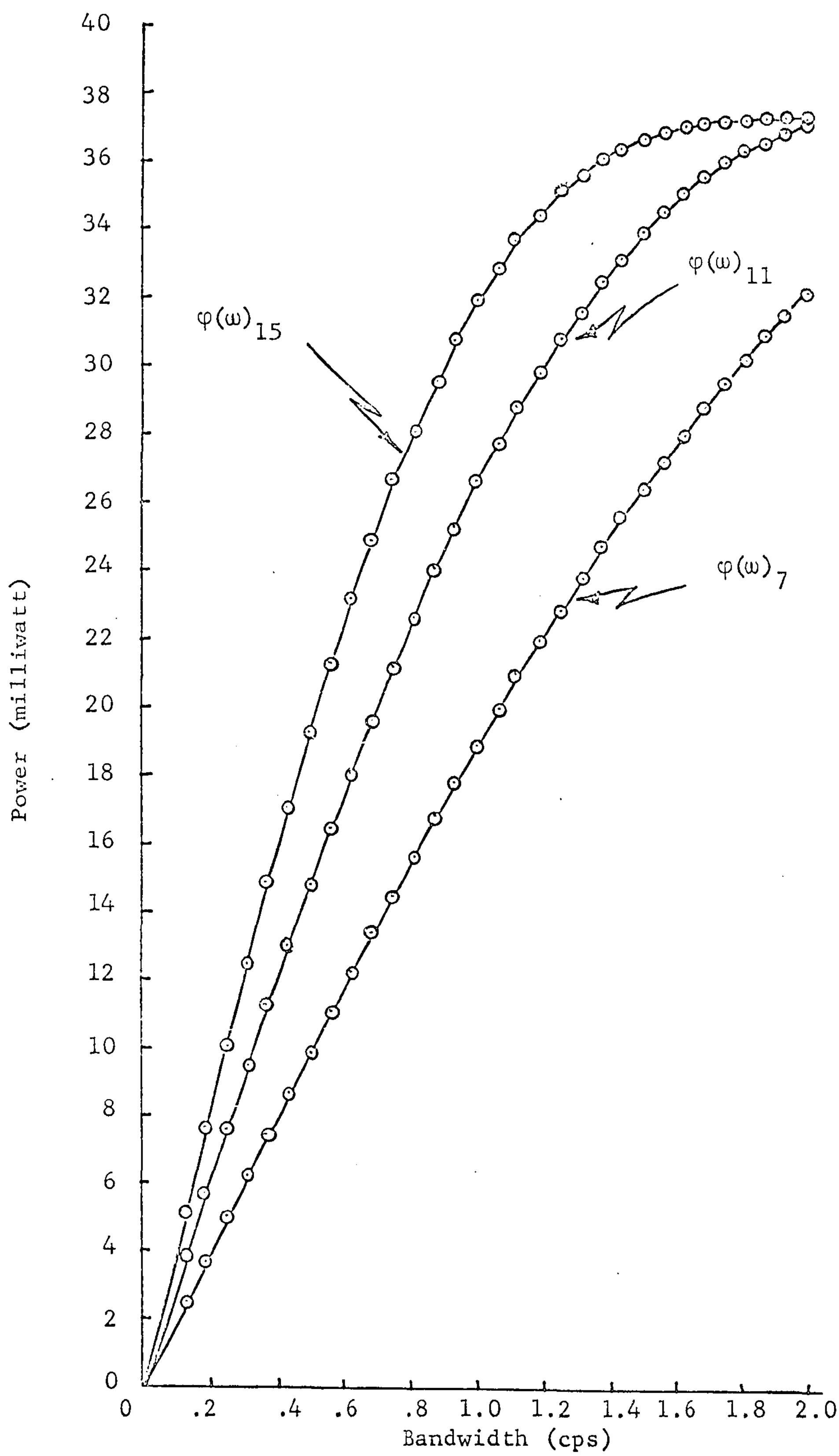

Finite-time spectral density (10^{-2}) (watt-sec)
0
1
2
0
74
74.2
74.4
74.6
74.8
75
75.2
75.4
75.6
75.8
76
Frequency (cps)
$\varphi(\omega)_{15}$
$\varphi(\omega)_{11}$
$\varphi(\omega)_7$

Fig. 20. Plots of the actual power contained in $\varphi(\omega)_7$, $\varphi(\omega)_{11}$, and $\varphi(\omega)_{15}$ versus bandwidth for a unit current for a center frequency of 75 cps


Power (milliwatt)
40
38
36
34
32
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
Bandwidth (cps)
φ(ω)15
φ(ω)11
φ(ω)7

It is assumed that this type of detection system is used. One problem that is encountered if this detection system is used is the fact that in a three-phase system all three lines of the system must be monitored by three separate receivers. This problem is overcome if a single current transformer is placed around all three conductors at the same time. If the system is balanced the 60 cps currents will cancel but the noise currents will add as the square root. This implies that the current signal needed is larger for detection than if each conductor is monitored separately. The disadvantage of using a larger current signal is overcome by the fact that only one receiver is needed to detect the current signal independent of which phase the signal is sent on.

Amount of current signal

If the bandwidth of the receiver is known it is possible to use equations 3 and 4 and Fig. 20 to determine the amount of signal current required for a load to draw that is switched on the line seven times. As before the load is connected 1/30 of a second and disconnected 1/30 of a second and repeated seven times. As before a detectable peak is located at 75 cps if the load is switched on and off at this rate. Equation 4 is a straight line approximation for the actual measurements of rms noise currents per one-half cycle bandwidth versus frequency. To find the approximate value of the rms noise current per one-half cycle bandwidth, i_N, for any frequency, f_x, if the total line current, I_T, is known use

$$i_N = (I_T/106)^{1/2} \left[-.325(10^{-3})(f_x - 60) + 23.2(10^{-3}) \right] \qquad (62)$$

The value of i_N squared is proportional to the power contained in one-half cycle bandwidth. The approximate current noise power contained in one cycle bandwidth is

$$P_N = (i_N^2/(1/2 \text{ cps}))((2)(1/2 \text{ cps})) = 2i_N^2 \tag{63}$$

Fig. 20 is a plot of the actual signal power versus bandwidth from zero to two cycles bandwidth about 75 cps for the switched load. The bottom curve is the one of interest for a unit current switched on and off the line seven times. Also the bandwidth of interest is one cycle. From the curve the point of interest is

$$P_s = A^2(18.95)(10^{-3}) \tag{64}$$

The ratio of interest is the signal-plus-noise to noise-power at the receiver. If this ratio is sixteen to one it is seen that

$$P_s = 15\ P_N$$

$$A^2(18.95)(10^{-3}) = 30\ i_N^2 \tag{65}$$

Using equations 62 and 65 it is possible to find the value of A or the peak value of the signal current at 75 cps for any I_T.

$$A = \left[(30\ i_N^2/(18.95)(10^{-3}))\right]^{1/2}$$
$$= (70.8)(10^{-3})\left[I_T\right]^{1/2} \tag{66}$$

For a one megawatt system the total current that flows at 120 volts for a completely resistive system is $I_T = 8333$ amperes. The peak value of the signal current is

$$A = (70.8)(10^{-3})\left[8333\right]^{1/2}$$
$$= 6.47 \text{ amperes} \tag{67}$$

Equation 67 shows the value of the signal current required to communicate with the source from any location in a one megawatt system to have a signal-plus-noise to noise-power ratio of sixteen to one. If the load is switched on and off the line more than seven times the signal current required would be less.

COMMUNICATING FROM THE SOURCE

Theory

Method of communication

To communicate from the source to any distant location it is necessary to place information on the line at a certain frequency and for a certain length of time. The length of time that the information must be on the line is found from the channel capacity as

$$C = W \log_2 (1 + P/N) \tag{68}$$

W is the bandwidth of the receiver and is in the order of one-half to one cps. P is the signal power and N is the noise power. For a receiver of one-half to one cps bandwidth and a signal-plus-noise to noise-power ratio of 16 to 1 the channel capacity is from 2 to 4 bits per second. The length of time that a single binary bit must be on the line is from ¼ to ½ seconds. The frequency of interest is anywhere between 70 and 110 cps because the rms noise voltage per bandwidth is known for this frequency range.

A signal generator placed in series with the source is needed to communicate with all locations if the signal generator supplies enough power at the frequency of interest. It is assumed that only one frequency, 90 cps, is used for the signal generator. The system under study was a three phase grounded wye system. A single phase is examined and it is assumed that the load is completely resistive (Fig. 21). The

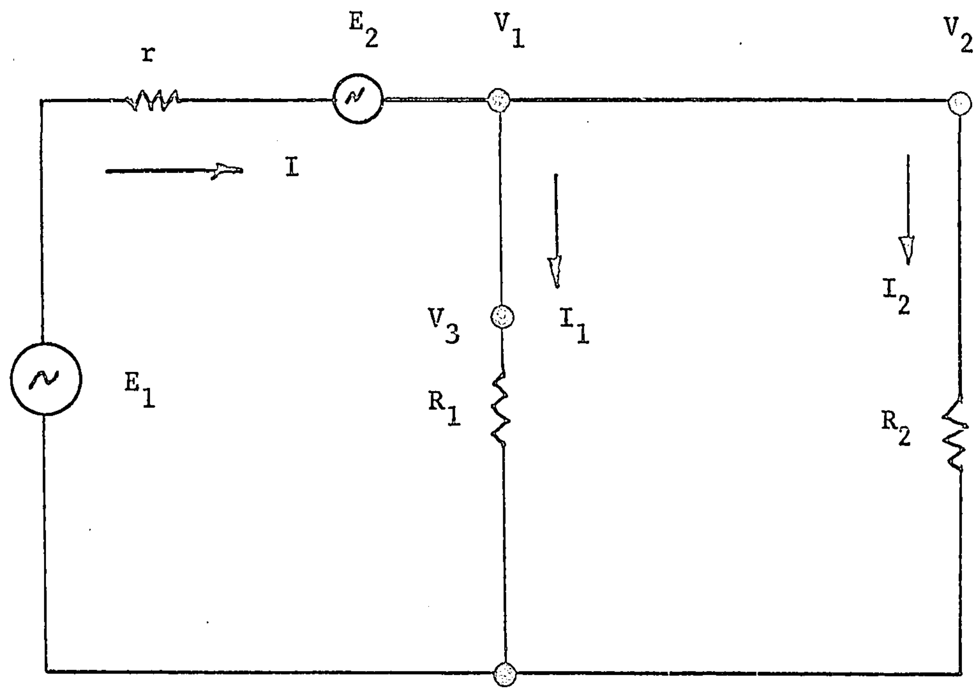


Fig. 21. A single phase of a three-phase system is shown. E_1 is the 60 cps source and E_2 is the voltage signaling source

circuit shown in Fig. 21 is a single phase system that does not show any transformer banks, actual length of the line, or the true number of loads on this phase. It is assumed that E_1 is the 60 cps source with internal resistance r, but the magnitude of E_1 is 120 volts instead of 2400 volts. This fact makes the magnitude of the current I very large. All of the external loads are combined into R_1 and R_2. The node V_1 is physically located at the power plant and nodes V_2 and V_3 are some distance from it.

If a single signal generator, E_2, is placed between r and V_1 physically it is possible to send information to either R_1, R_2, or R_1 and R_2 provided the power supplied is larger than the noise generated by the source E_1. To find the noise power supplied by E_1 it is necessary to use Fig. 5 to determine the amount of power contained in a region between two frequencies f_1 and f_2. Fig. 5 is a plot of rms noise voltage per one-half cycle bandwidth versus frequency. The bandwidth used to obtain Fig. 5 was a one-half cycle tuning fork. If the voltage is squared a plot proportional to noise power per one-half cycle bandwidth versus frequency is obtained. The area under this curve from f_1 to f_2 is proportional to the noise power supplied by E_1. Let the average value of rms noise voltage between f_1 and f_2 be E_N. The noise power, P_{Ni}, supplied by E_1 to R_i is

$$P_{Ni} = \int_{f_1}^{f_2} \frac{E_N^2}{R_i} \, df = \frac{E_N^2}{R_i} (f_2 - f_1) \tag{69}$$

The quantity E_N^2/R_i is the average noise power per one-half cycle bandwidth between f_1 and f_2. The bandwidth of the receiver is one cycle. The noise power supplied to R_i by E_1 is

$$P_{Ni} = \frac{(E^2{}_N/R_i)}{(\frac{1}{2}\text{ cps})} \cdot 2\ (\tfrac{1}{2}\text{ cps}) = 2E^2{}_N/R_i \tag{70}$$

It is noted that as the receiver bandwidth increases so does the noise power into the receiver increase.

The amount of power supplied to R_i by E_1 is

$$P_{1i} = \frac{V_2^2}{R_i} = \frac{E_1^2\,(R_1R_2/(R_1 + R_2))^2}{(r + (R_1R_2/(R_1 + R_2)))^2}\ 1/R_i \tag{71}$$

Because r is much less than R_1 or R_2 the result is approximately

$$P_{1i} = \frac{V_2^2}{R_i} \simeq \frac{E_1^2}{R_i} \tag{72}$$

The total power supplied by E_1 is approximately

$$P_{1T} \simeq E_1^2\,(R_1 + R_2)/(R_1R_2) \tag{73}$$

The amount of signal power supplied to R_i by E_2 is approximately

$$P_{2i} \simeq E_2^2\,/\,R_i \tag{74}$$

The total signal power supplied by E_2 is approximately

$$P_{2T} \simeq E_2^{\,2}\,(R_1 + R_2)/(R_1R_2) \tag{75}$$

The ratio of P_{Ni}/P_{1i} is of interest because as the total input power supplied increases or decreases this ratio is the same for a constant voltage source provided r is very small. This ratio is the fraction of noise power to actual power supplied to R_i by E_1. As the total input power increases so must the input power supplied by E_2 increase. The power supplied by E_2 is a linear function of the total input power, P_{1T}. P_s is equal to the product of the total input power and the ratio of P_{N2}/P_{12} supplied to R_2.

$$P_s = P_{1T}(P_{N2}/P_{12}) \tag{76}$$

P_s is the total signal power required by E_2 to have a signal-to-noise power ratio of one if the detection system used has a one cycle bandwidth. It is noted that P_{N2} is a function of the bandwidth from equation 70. As the bandwidth of the receiver is increased so must the total signal power supplied increase. It is also assumed that all of the power supplied by E_2 is contained in this one cycle bandwidth.

<u>Power spectral density</u>

From the definition of channel capacity, equation 59, it is possible to determine the theoretical minimum length of time that one bit of information must be present in order to communicate from one location to another. T_2 is the time that one bit of information must be present and $y(t) = B \sin(\omega_2 t)$ is the information bit for T_2 seconds. It is necessary to look at the finite-time spectral density of y(t) to obtain the amount of power that is supplied for a given bandwidth. The finite-time spectral density is defined as

$$\varphi(\omega) = (1/T_2) \left| F(\omega) \right|^2 \tag{77}$$

The Fourier transform of y(t) is

$$\begin{aligned} \underline{F}\left[y(t)\right] &= \int_{-T_2/2}^{T_2/2} y(t)\, e^{-j\omega t}\, dt \\ &= \int_{-T_2/2}^{T_2/2} B \sin(\omega_2 t) \cos(\omega t)\, dt \\ &\quad - j \int_{-T_2/2}^{T_2/2} B \sin(\omega_2 t) \sin(\omega t)\, dt \end{aligned} \tag{78}$$

The first term in equation 78 is zero because y(t) is odd.

$$\begin{aligned} \underline{F}\left[y(t)\right] &= \left[B/j(\omega - \omega_2)\right] \sin((\omega - \omega_2)T_2/2 \\ &\quad - \left[B/j(\omega + \omega_2)\right] \sin((\omega + \omega_2)T_2/2 \end{aligned} \tag{79}$$

The complex conjugate of equation 79 is

$$\underline{F}^* \left[y(t) \right] = \left[Bj/(\omega - \omega_2) \right] \sin((\omega - \omega_2)T_2/2$$

$$- \left[Bj/(\omega + \omega_2) \right] \sin((\omega + \omega_2)T_2/2 \qquad (80)$$

The finite-time spectral density of y(t) is

$$\begin{aligned} \varphi(\omega) &= (1/T_2)\, \underline{F}\left[y(t) \right] \underline{F}^* \left[y(t) \right] \\ &= \left[B^2/(\omega - \omega_2)^2 T_2 \right] \sin^2((\omega - \omega_2)T_2/2) \\ &+ \left[B^2/(\omega + \omega_2)^2 T_2 \right] \sin^2((\omega + \omega_2)T_2/2) \\ &- \frac{\left[2B^2 \right] \sin((\omega - \omega_2)T_2/2 \; \sin((\omega + \omega_2) T_2/2}{T_2(\omega - \omega_2)(\omega + \omega_2)} \end{aligned} \qquad (81)$$

Equation 81 is completely general for any $y(t) = B \sin(\omega_2 t)$ that is on for the time T_2. This finite-time spectral density function is only good for the time T_2 that y(t) is not zero. As T_2 is increased the shape of $\varphi(\omega)$ becomes narrower and in the limit as T_2 goes to infinity $\varphi(\omega)$ becomes an impulse function.

Mathematical Model

Computer program

A computer program was written to find the finite-time spectral density of a 1/4, 1/2, 3/4, and 1 seconds information bit, y(t), that is sent from the source. The function y(t) is a 90 cps sinusoid that is placed on the line for T_2 seconds. The finite-time spectral density for y(t) is given in equation 81. This equation is general for any ω_2 or T_2. For $\omega_2 = 2\pi(90)$ radians per second and $T_2 = 1/4$ or $3/4$ seconds equation 81 becomes

$$\varphi(\omega) = (A^2/T_2)\left[(4\omega_2^{\,2}/(\omega^2 - \omega_2^{\,2})^2)\right] \cos^2(\omega T_2/2) \tag{82}$$

For $\omega_2 = 2\pi(90)$ and $T_2 = 1/2$ or 1 second equation 81 becomes

$$\varphi(\omega) = (A^2/T_2)\left[(4\omega_2^{\,2}/(\omega^2 - \omega_2^{\,2})^2)\right] \sin^2(\omega T_2/2) \tag{83}$$

A Fortran computer program evaluates the finite-time spectral density of y(t) for $T_2 = 1/4$, 1/2, 3/4, and 1 seconds. Equations 82 and 83 are used for $T_2 = 1/4$ or 3/4 and 1/2 or 1 seconds respectively.

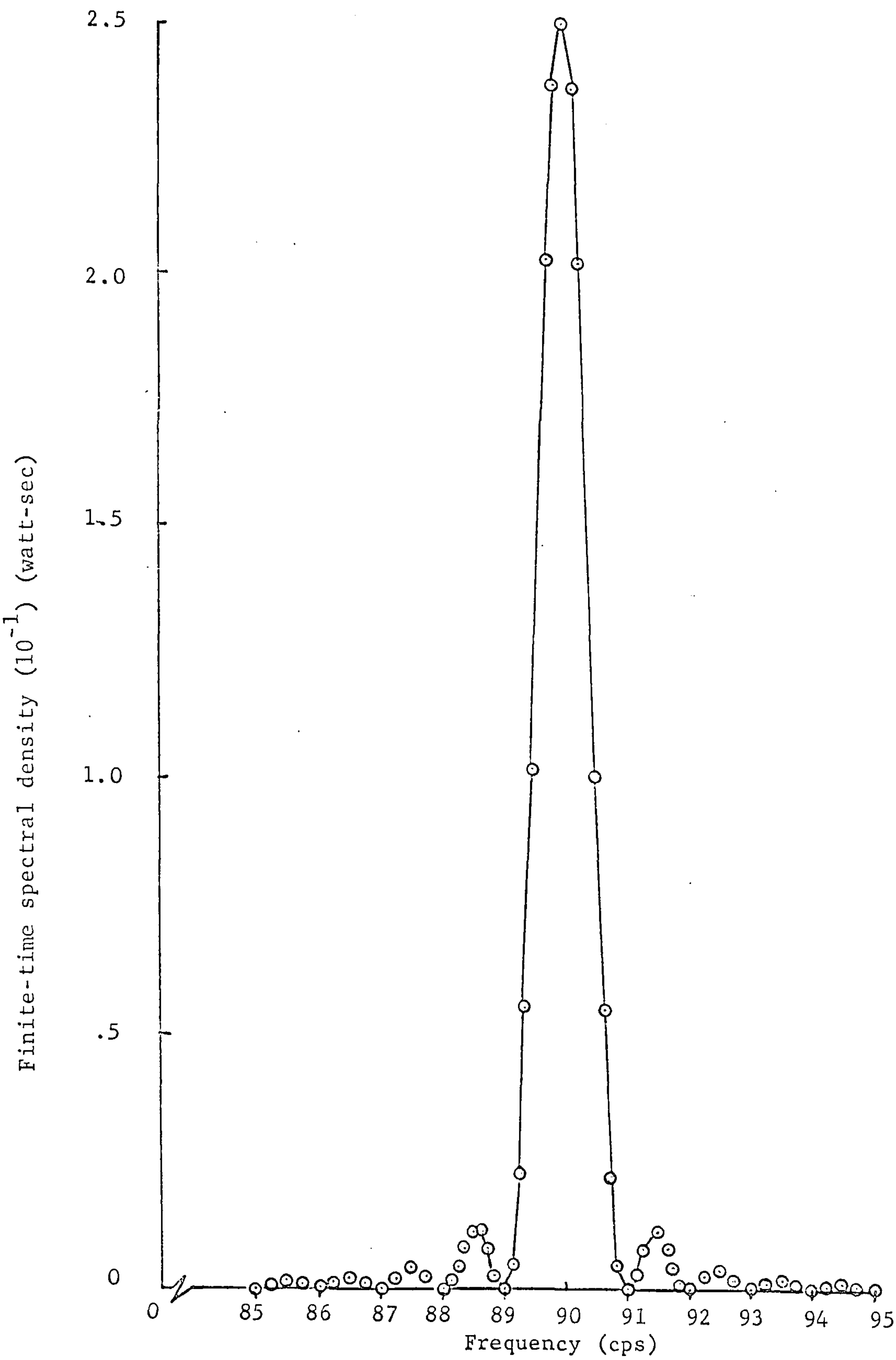
The actual plots of the finite-time spectral density of y(t), between 85 and 95 cps, for T_2 equal to 1/4, 1/2, 3/4, and 1 seconds are shown in Figures 25, 24, 23, and 22 respectively. Fig. 26 shows all four functions on the same graph between 89 and 91 cps. Note that as the time, T_2, increases the maximum amplitude of $\varphi(\omega)$ increases. Fig. 27 shows the actual power for a unit voltage as a function of the bandwidth centered about 90 cps.

Detecting and Amount of Voltage Signal

Method

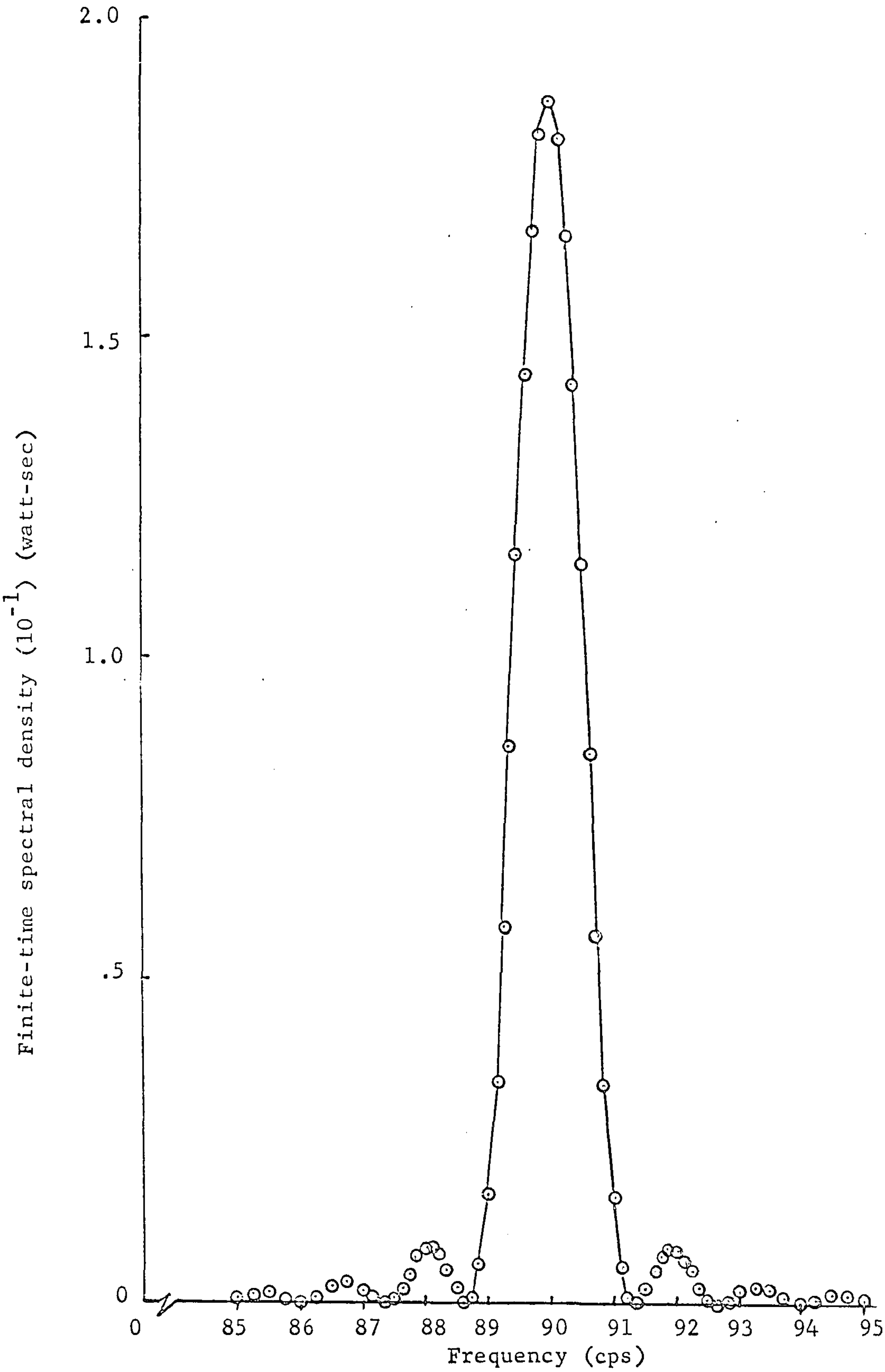
The voltage signal is placed on the line with a current transformer,

Fig. 22. Plot of finite-time spectral density versus frequency for $y(t)$ when the total time that $y(t)$ is on is one second



2.5
2.0
1.5
1.0
.5
0
Finite-time spectral density (10^{-1}) (watt-sec)
0
85
86
87
88
89
90
91
92
93
94
95
Frequency (cps)

Fig. 23. Plot of finite-time spectral density versus frequency for y(t) when the total time that y(t) is on is 3/4 of a second



2.0
1.5
1.0
.5
0
Finite-time spectral density (10^{-1}) (watt-sec)
0
85
86
87
88
89
90
91
92
93
94
95
Frequency (cps)

Fig. 24. Plot of finite-time spectral density versus frequency for $y(t)$ when the total time that $y(t)$ is on is 1/2 of a second

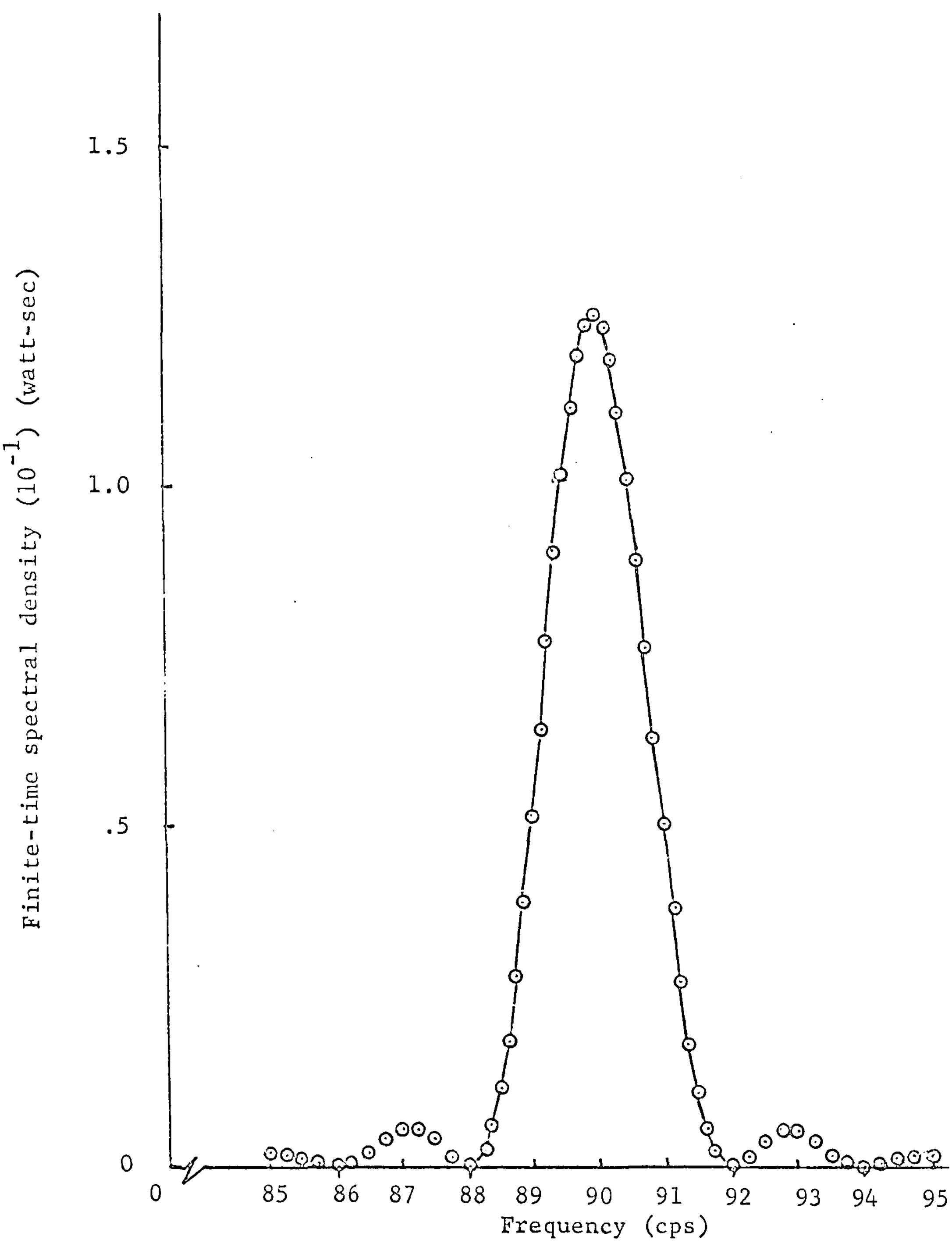
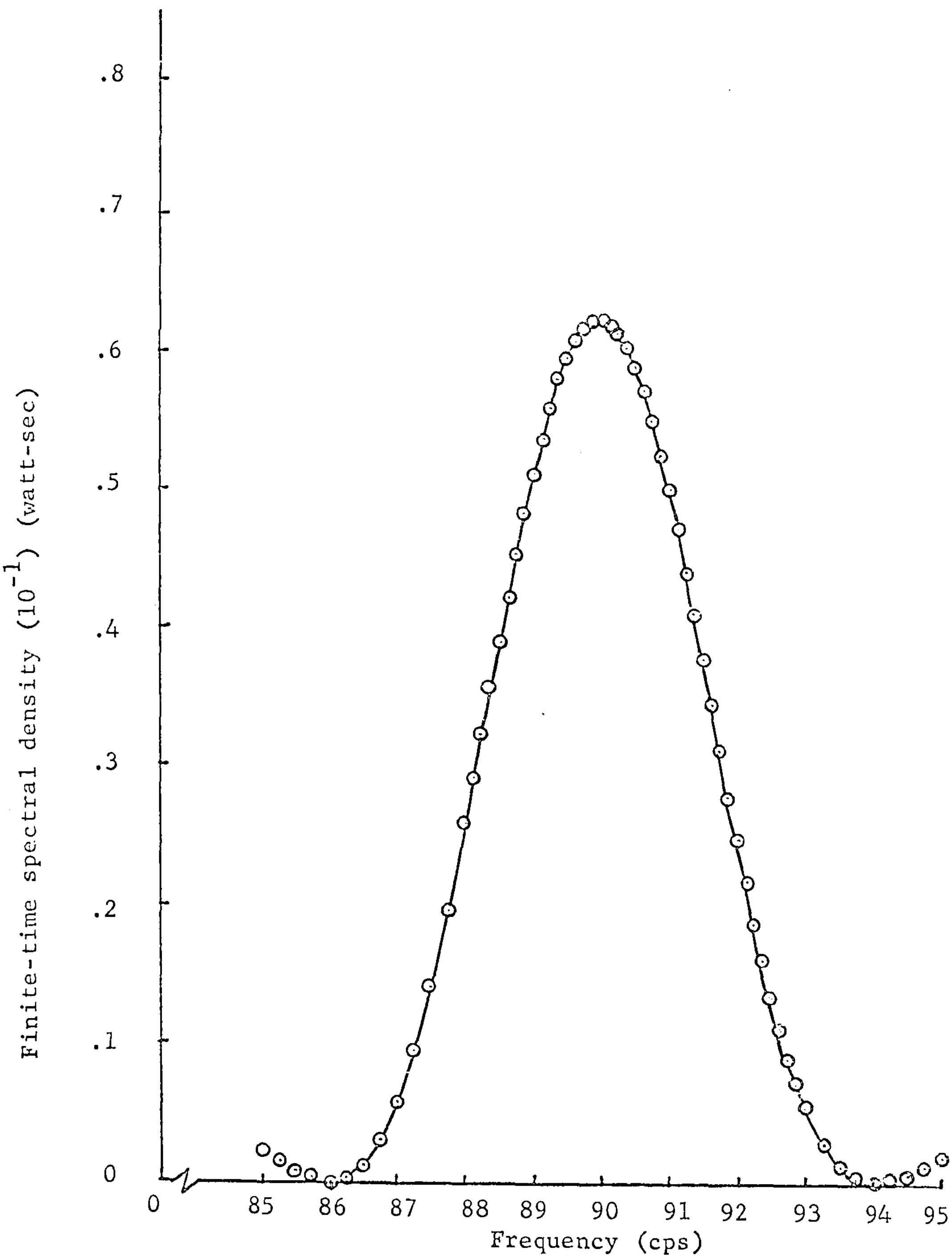
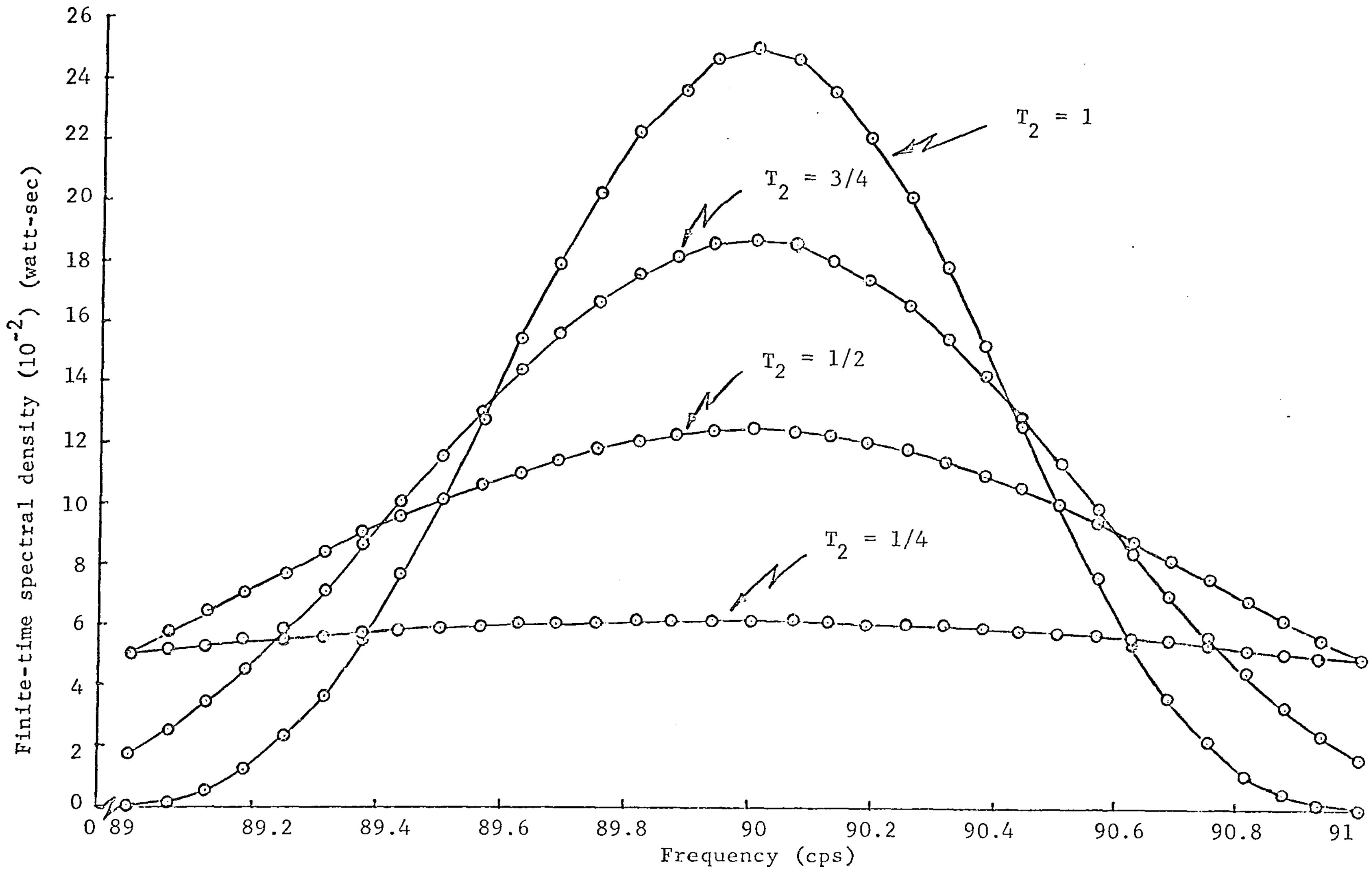

Finite-time spectral density (10^{-1}) (watt-sec)
1.5
1.0
.5
0
0
85
86
87
88
89
90
91
92
93
94
95
Frequency (cps)

Fig. 25. Plot of finite-time spectral density versus frequency for $y(t)$ when the total time that $y(t)$ is on is 1/4 of a second



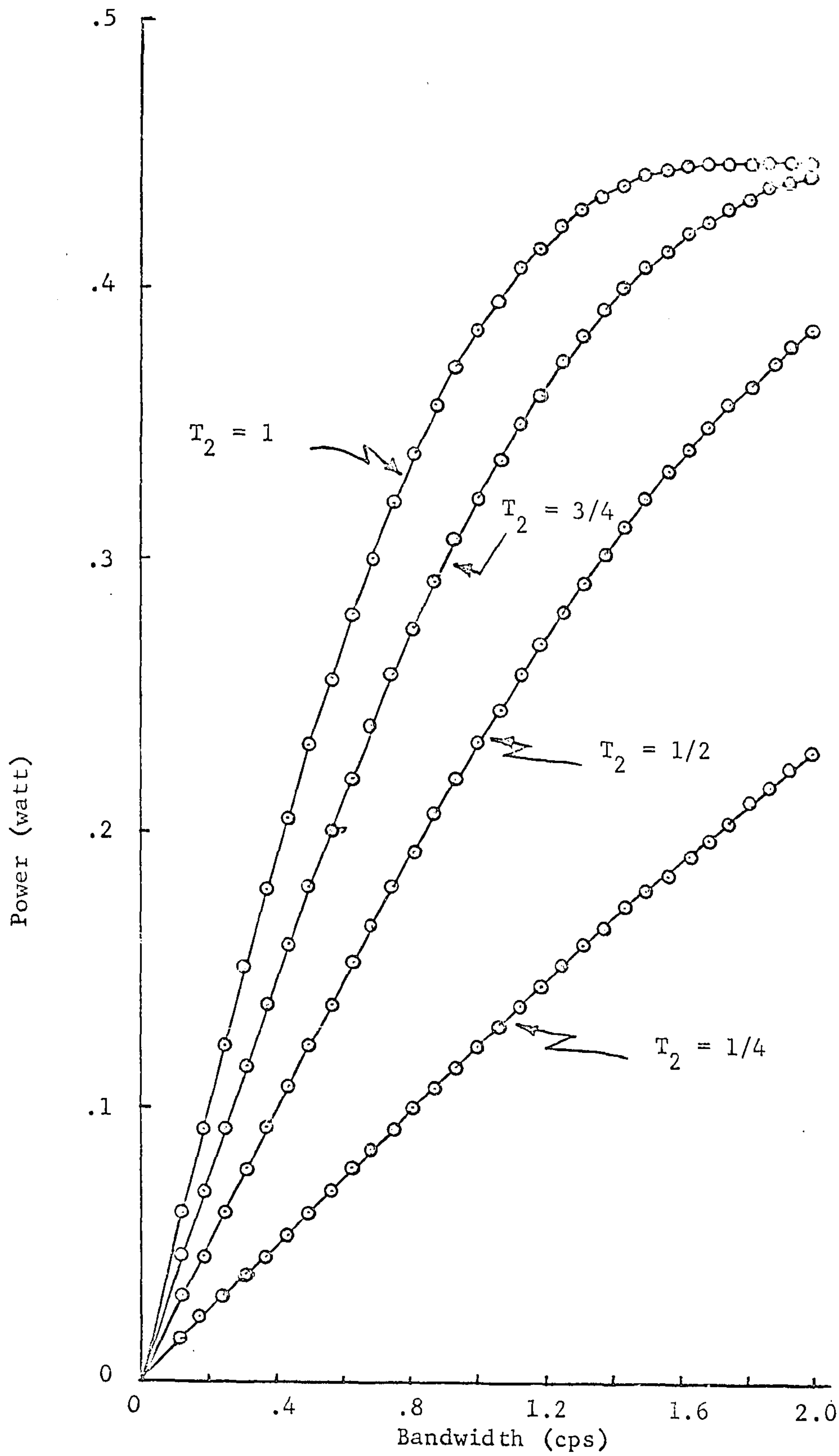
Finite-time spectral density (10^{-1}) (watt-sec)
.8
.7
.6
.5
.4
.3
.2
.1
0
0
85
86
87
88
89
90
91
92
93
94
95
Frequency (cps)

Fig. 26. Plots of finite-time spectral densities versus frequency for y(t) when the total time that y(t) is on is 1, 3/4, 1/2, and 1/4 of a second around the center frequency of 90 cps



Finite-time spectral density (10^{-2}) (watt-sec)
0
2
4
6
8
10
12
14
16
18
20
22
24
26
0
89
89.2
89.4
89.6
89.8
90
90.2
90.4
90.6
90.8
91
Frequency (cps)
T_2 = 1
T_2 = 3/4
T_2 = 1/2
T_2 = 1/4

Fig. 27. Plots of the actual power contained in $\varphi(\omega)_1$, $\varphi(\omega)_{3/4}$, $\varphi(\omega)_{1/2}$, and $\varphi(\omega)_{1/4}$ versus bandwidth for a unit voltage at a center frequency of 90 cps


.5
.4
.3
.2
.1
0
Power (watt)
0
.4
.8
1.2
1.6
2.0
Bandwidth (cps)
T_2 = 1
T_2 = 3/4
T_2 = 1/2
T_2 = 1/4

signal generator, and a power amplifier. The magnetic material of the current transformer is placed completely around one of the three conductors at the point where the three conductors supply power to a fraction of the total load on the system. A disadvantage arises in supplying power to only one conductor in a three-phase system. There are four possible ways that this power may be transformed in the transformer banks that are in a three-phase system, delta-delta, wye-wye, wye-delta, and delta-wye.

For the delta-delta or the delta-wye system that is supplied voltage signal power on only one conductor the power divides equally into only two of the three loads if the system is balanced. To communicate with the third load it is necessary to supply the power to one of the other two conductors at the power plant. This problem is not solved by supplying power to two conductors in phase at the same time because only two of the three loads can be reached if this is done. If all three conductors are supplied power, that is in phase, at the same time the results are that no signal power can reach the three loads. The only way that all three loads can be reached at any time is to supply all three conductors with power but have each signal spaced 120 degrees apart.

For the wye-wye system it is necessary to supply voltage signal power to all three conductors at the same time in order to communicate with any location. It is not necessary to have each signal spaced 120 degrees apart as in the above example.

For the wye-delta system it is necessary to supply voltage signal power to each of the three conductors spaced 120 degrees apart to communicate with any location in the system. If one or two of these conductors are supplied power which is in phase this power is transformed such that 2/3 of the power is supplied to one phase and 1/3 of the power is supplied to the other two phases. If all three conductors are supplied power from a single phase source the results are that no signal power can reach the three loads. It is seen from the above that it is necessary to supply three phase voltage signal power to a general system to communicate with any remote location.

Amount of voltage signal

From equation 76 it is seen that the total power supplied by E_2 is

$$P_s = P_{1T}(P_{Ni}/P_{1i}) \tag{84}$$

for a signal-to-noise power ratio of one. P_{Ni} is the noise power supplied to R_i in a certain bandwidth, P_{1i} is the 60 cps power supplied to R_i by E_1, and P_{1T} is the total power supplied to the system by E_1. It is assumed that all of the signal power is contained in a very small bandwidth. Fig. 27 shows the actual signal power versus bandwidth centered at 90 cps for different signal times, T_2. The actual power required is seen to be a function of T_2 and the bandwidth of the receiver. Assume a one cycle bandwidth receiver. Let the total 60 cps input power be one megawatt, let T_2 be one second, and let E_1, Fig. 21, be 120 volts rms. T_2 is the actual time that each information bit is on the line.

The noise power, P_{Ni}, supplied by E_1 to R_i is

$$P_{Ni} = \int_{f_1}^{f_2} E_N^{\,2}/R_i \cdot df = E_N^{\,2}(f_2 - f_1)/R_i \tag{85}$$

The quantity $E_N^{\,2}/R_i$ is the average noise power per one-half cycle bandwidth between f_1 and f_2. The value of E_N is found from Fig. 5 to be 1.47 millivolts at 90 cps for the actual measured value. For a one cycle bandwidth receiver the noise power supplied to R_i is

$$P_{Ni} = \frac{(E_N^{\,2}/R_i)}{(1/2 \text{ cps})}\ 2(1/2 \text{ cps}) = 2E_N^{\,2}/R_i \tag{86}$$

The 60 cps power supplied to R_i by E_1 is

$$P_{1i} = (120)^2/R_i \tag{87}$$

If all the signal power is contained in a one cycle bandwidth and a signal-to-noise power ratio of 15 to 1 is needed the total signal power required by E_2 is

$$P_{s2} = 15\ P_s = 15\ P_{1T}(P_{Ni}/P_{1i}) \tag{88}$$

It is seen from Fig. 27 that for a one second information bit and for a bandwidth of one cycle the actual power contained is .385/.5 of the total instantaneous power. The actual power, P_{sT}, that must be placed on the line by E_2 for a signal-to-noise power ratio of 15 to 1 if the information bit is one second in length and the bandwidth is one cycle is

$$
\begin{aligned}
P_{sT} &= (.5/.385)\ P_{s2} \\
&= 1.3\ P_{s2} \;=\; 19.48\ P_s \qquad (89) \\
&= 5.85(10^{-3}) \text{ watts}
\end{aligned}
$$

Equation 89 shows the actual signal power required by E_2 to supply to a one megawatt system is only 5.85 milliwatts. This amount of signal power is all that is required in order to communicate with any location in the system. The amount of signal power required is a function of both the time that the signal is sent, T_2, and the bandwidth of the receiver.

Bandwidth versus rise time

To obtain an approximation of the rise time of an information bit a perfect inductor, L, and capacitor, C, are placed in series with a resistor, R, to control the bandwidth of the series resonant circuit. The input signal is placed across the entire circuit and the output is taken across the resistor which is grounded. The transfer function is

$$
\begin{aligned}
Y(s) &= e_{out}(s)/e_{in}(s) = RCs/(LC\ s^2 + RC\ s + 1) \\
&= (R/L)\ s/(s^2 + (R/L)\ s + (1/LC)) \qquad (90)
\end{aligned}
$$

The resonant frequency, ω_r, is

$$\omega_r = \left[1/LC\right]^{1/2} \tag{91}$$

The figure of merit, or Q, of a series resonant circuit is

$$Q = \omega_r L/R = f_r/B \tag{92}$$

B is the bandwidth of the series resonant circuit measured in cps. By solving equation 92 for R/L and substituting it into equation 90 it is shown that Y(s) is a function of the bandwidth B.

$$Y(s) = e_{out}(s)/e_{in}(s) = (2\pi B)\ s \ / \ (s^2 + (2\pi B)\ s + \omega_r^2) \tag{93}$$

The input voltage, $e_{in}(t)$, is

$$e_{in}(t) = A \sin(\omega_r t) \tag{94}$$

The Laplace transform of $e_{in}(t)$ is

$$e_{in}(s) = \int_0^\infty e_{in}(t)\ e^{-st}\ dt$$

$$= A\omega_r/(s^2 + \omega_r^2) \tag{95}$$

The output voltage, $e_{out}(t)$, is found by taking the inverse Laplace transform of the product of Y(s) and $e_{in}(s)$.

$$e_{out}(t) = L^{-1}\left[Y(s)\ e_{in}(s)\right]$$

$$= L^{-1}\left[A\omega_r\ (2\pi B)\ s/(s^2 + \omega_r^2)(s^2 + (2\pi B)\ s + \omega_r^2)\right] \tag{96}$$

$$= A\sin(\omega_r t) - \frac{(Ae^{-\pi Bt})\ \sin\left[\omega_r(1 - (\pi B/\omega_r)^2)^{1/2}t\right]}{\left[1 - (\pi B/\omega_r)^2\right]^{1/2}}$$

The following approximation is made for $f_r = 90$ cps

$$\left[1 - (\pi B/\omega_r)^2\right]^{1/2} \cong 1 - (\pi B/\omega_r)^2(1/2) = 1 - B^2/64800$$

$$= 1 - z \tag{97}$$

where z is $(B^2/64800)$.

By substituting equation 97 into equation 96, it is possible to obtain

$$e_{out}(t) = A\sin(\omega_r t) - \left[A\ e^{-\pi Bt}/(1 - (\pi B/\omega_r)^2)^{1/2}\right]\sin(\omega_r(1 - z)t)$$

$$= A\left[1 - e^{-\pi Bt}\cos(\omega_r zt)/(1 - (\pi B/\omega_r)^2)^{1/2}\right]\sin(\omega_r t)$$

$$+ A\left[e^{-\pi Bt}\sin(\omega_r zt)/(1 - (\pi B/\omega_r)^2)^{1/2}\right]\cos(\omega_r t) \tag{98}$$

Let k_1 and k_2 be the coefficients of $\sin(\omega_r t)$ and $\cos(\omega_r t)$ respectively. Note that k_1 and k_2 are both functions of time. The output voltage is now

$$
\begin{aligned}
e_{out}(t) &= k_1 \sin(\omega_r t) + k_2 \cos(\omega_r t) \\
&= \left[k_1^2 + k_2^2\right]^{1/2} \sin\left[\omega_r t + \tan^{-1}(k_2/k_1)\right]
\end{aligned}
\tag{99}
$$

The positive envelope of $e_{out}(t)$ is plotted in Fig. 28 to show the rise time of $e_{out}(t)$ as a function of the different bandwidths of the series resonant circuit.

The output voltage from a narrow band receiver will act approximately as $e_{out}(t)$ does for a simple series resonant circuit. As the bandwidth of the receiver is made smaller the output voltage requires more time to build up to its final value. The times required to build up to 90 per cent of its final value are .18, .24, .37, .74, .99, and 1.5 seconds for bandwidths of 4, 3, 2, 1, 3/4, and 1/2 cycles respectively. This example shows the relationship between the rise time of the output and the bandwidth of a series resonant circuit. The rise time required by a narrow band filter like a tuning fork is approximately the same as this series resonant circuit.

Figures 29 and 30 show actual signals that were detected on a 60 cps transmission line. The signal-plus-noise to noise-power ratio was about 300 to 1 so the rise time could be measured. The rise times of these signal pulses were 1.5 seconds.

Fig. 28. Plots of the envelope of $e_{out}(t)$ versus frequency for different bandwidths

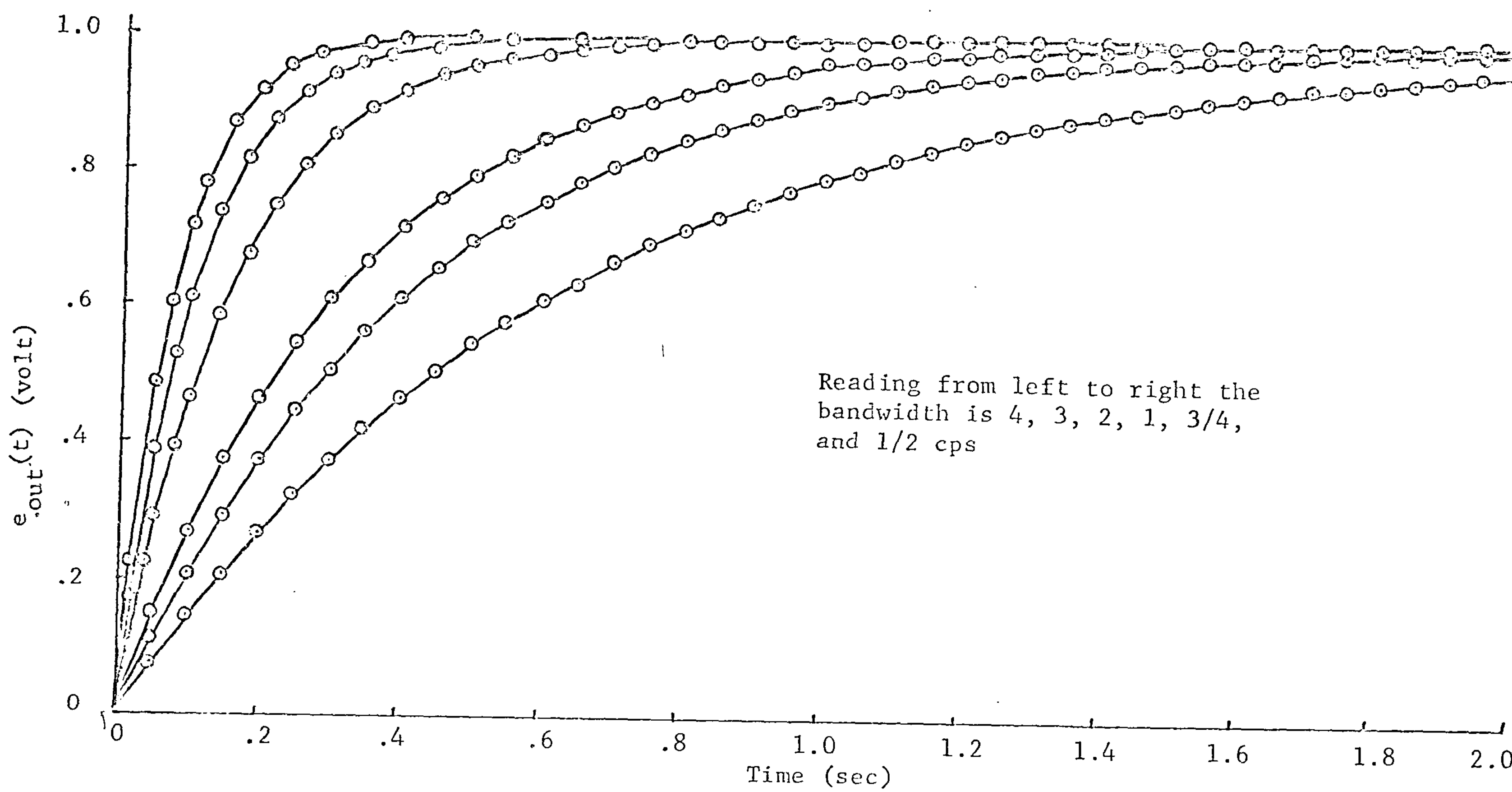
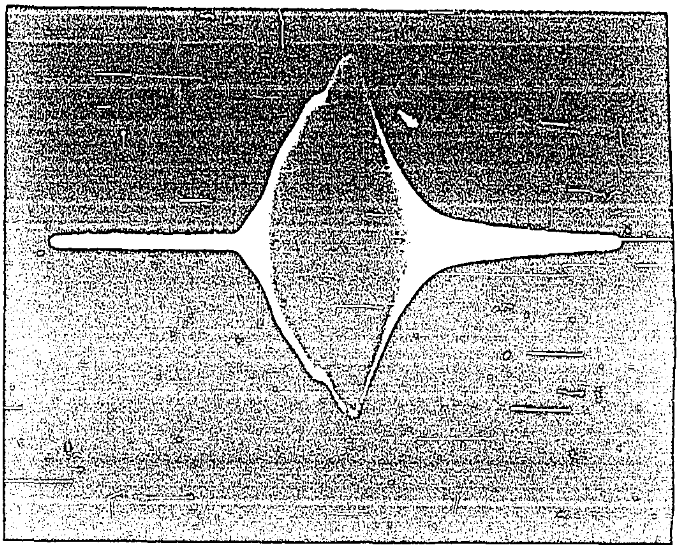
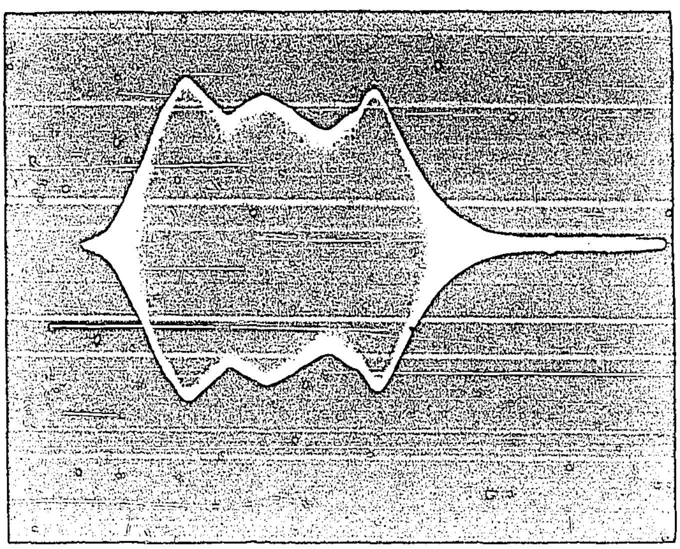

1.0
.8
.6
.4
.2
0
$e_{out}(t)$ (volt)
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
Time (sec)
Reading from left to right the bandwidth is 4, 3, 2, 1, 3/4, and 1/2 cps

Fig. 29. Typical photo of a 2 second signal on a 60 cps transmission line after it has passed through a narrow band receiver. The vertical scale is 100 millivolts per centimeter and the horizontal scale is 1 second per centimeter

Fig. 30. Typical photo of a 5 second signal on a 60 cps transmission line after it has passed through a narrow band receiver. The vertical scale is 100 millivolts per centimeter and the horizontal scale is 1 second per centimeter





EXPERIMENTAL SIGNALING EQUIPMENT

The actual signal was placed on the line in the basement of the Electrical Engineering Building with a current transformer, signal generator, and a power amplifier. Fig. 31 shows the signal generator, power amplifier, current transformer, rms voltmeter, and an ammeter at the left, center, upper right, lower right, and center right of the photograph respectively. Two of the three-phase power transformers can be seen in the photograph. The basement was chosen because all the main input power to the building goes through these power transformers.

The current transformer was a clamp on type that was placed completely around the conductor. Open circuit tests on the current transformer were performed to determine the shunt resistance, r_m, and the magnetizing reactance, x_m. The shunt resistance had a value of 13.5 ohms and the magnetizing reactance had a value of 7.68 ohms. The turns ratio of the current transformer was known to be 40 to 1 so it was possible to determine the actual power that was placed on the line.

A true reading rms voltmeter was used to measure the voltage on the current transformer secondary. The initial voltage, V_i, was the actual measure of the 60 cps component of the voltage. The final voltage, V_f, with the actual signal applied was the true rms voltage of both the initial 60 cps component and the 80 cps signal component. The difference between V_f and V_i was the actual 80 cps signal component added to the current transformer secondary. Once the magnitude of the 80 cps signal component was known it was possible to find the amount of power supplied to r_m and the actual power supplied to the line. The line current was 155 amperes so the load was assumed to be completely resistive and its value was $R_L = .775$ ohms. The value was transformed to the secondary of the current transformer as $R_{LT} = 1240$. The equivalent circuit of the current transformer was three elements in parallel r_m, 1_m and R_{LT}. The power delivered to R_{LT} can be shown to be equal to

$$P_{R_{LT}} = P_{r_m}(r_m/R_{LT})$$

$$= (V_f - V_i)^2/R_{LT} \tag{100}$$

$$= 8.06(10^{-6}) \text{ watts}$$

The signal-to-noise ratio with this amount of added power was poor but the signal was detectable. Using the experimental results of Fig. 5 to predict the amount of signal power required at 80 cps and equation 76 it takes

$$P_s = P_{IT}(P_{N2}/P_{12})$$

$$= (1.86(10^{+4}))(2.2)^2(10^{-6})/(120)^2 \tag{101}$$

$$= 6.26(10^{-6}) \text{ watts}$$

of signal power to have a signal-to-noise power ratio of one. The amount of predicted power required and the actual power supplied are in agreement.

The actual signal was detected in the research area of the Electrical Engineering Building using a twin-T filter to block the 60 cps component, a HP 302A wave analyzer, an amplifier, and a tuning fork. The detecting equipment was some 60 yards from the basement and the transmitting equipment. Fig. 32 shows the equipment that was used to detect the actual signal and most of the equipment used in the noise measurements. The twin-T filter, HP 302A, amplifier, and tuning fork are shown in the center, right, top and left of center, and top and right of center of the photograph respectively. The equipment at the left, bottom and left of center, and bottom and right of center are the power supply, amplifier and mixer, and another tuning fork respectively that were used in the noise measurements. All of the equipment used for the noise measurements is not shown in this photograph.

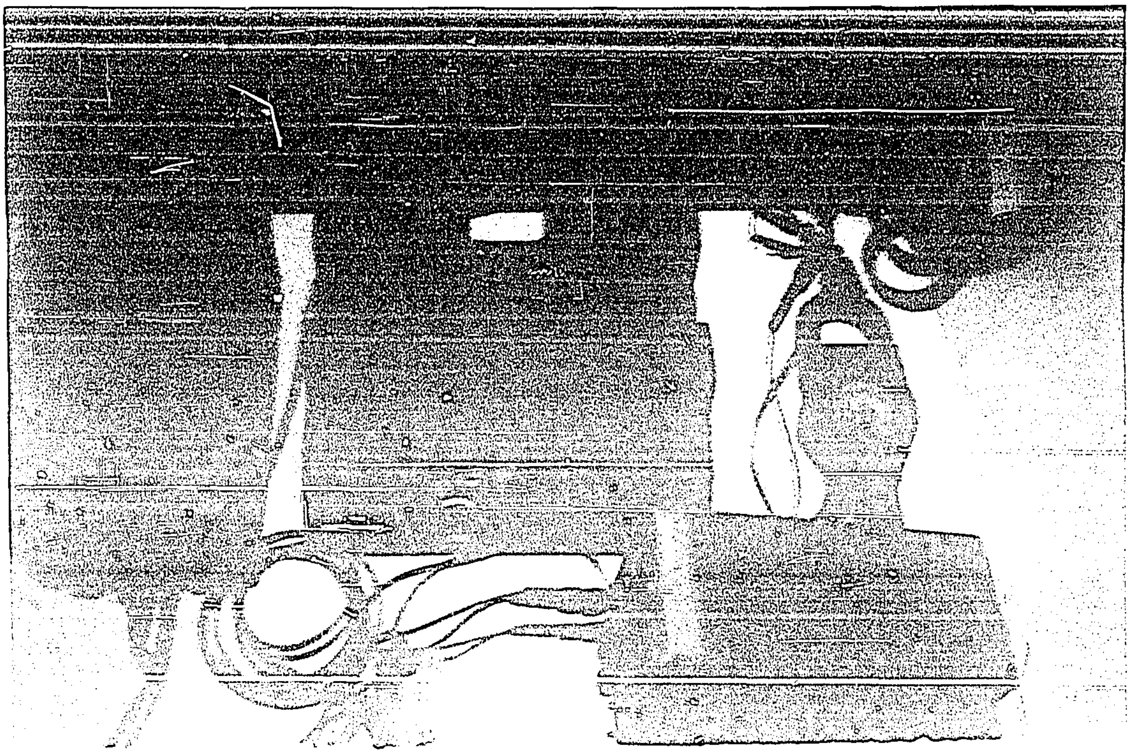
The incoming signal into the HP 302A wave analyzer was first attenuated to a level which prevents overloading the input amplifier. After this was amplified, a low-pass filter rejected any possible 100 kc component. Frequency conversion of the input signal to the 100 kc intermediate frequency occurs in the balanced modulator, which was driven by a 100 kc to 150 kc local oscillator. The high selectivity of the instrument was obtained by cascading two crystal-tuned IF filters. The local oscillator signal was mixed with the 100 kc IF signal in the output modulator. The lower sideband component was identical with the original input signal frequency and was available at the output jack.

The output from the HP 302A was fed into an amplifier that drove a small coil at 80 cps. The small coil was a U-shaped iron piece that was placed along one side of the tuning fork. The U-shaped iron did not physically touch the tuning fork but the magnetic force on the tuning fork drove it. A pick-up coil was wound on a U-shaped permanent magnet placed on the other side of the tuning fork to detect the vibrations.

A signal was placed on the line that had a signal-plus-noise to noise-power ratio of 16 to 1. The signal power was $93.9(10^{-6})$ watts. Fig. 33 shows this signal-plus-noise that was visible on an oscilloscope. The signal was placed on the line for two seconds and removed for three seconds and repeated.

Fig. 31. The equipment used to communicate from the source to any remote location in the field. This equipment was located in the basement of the Electrical Engineering Building

Fig. 32. The equipment used to detect the actual signal sent from the basement and most of the equipment used for the noise measurements. This equipment was located in the research area of the Electrical Engineering Building



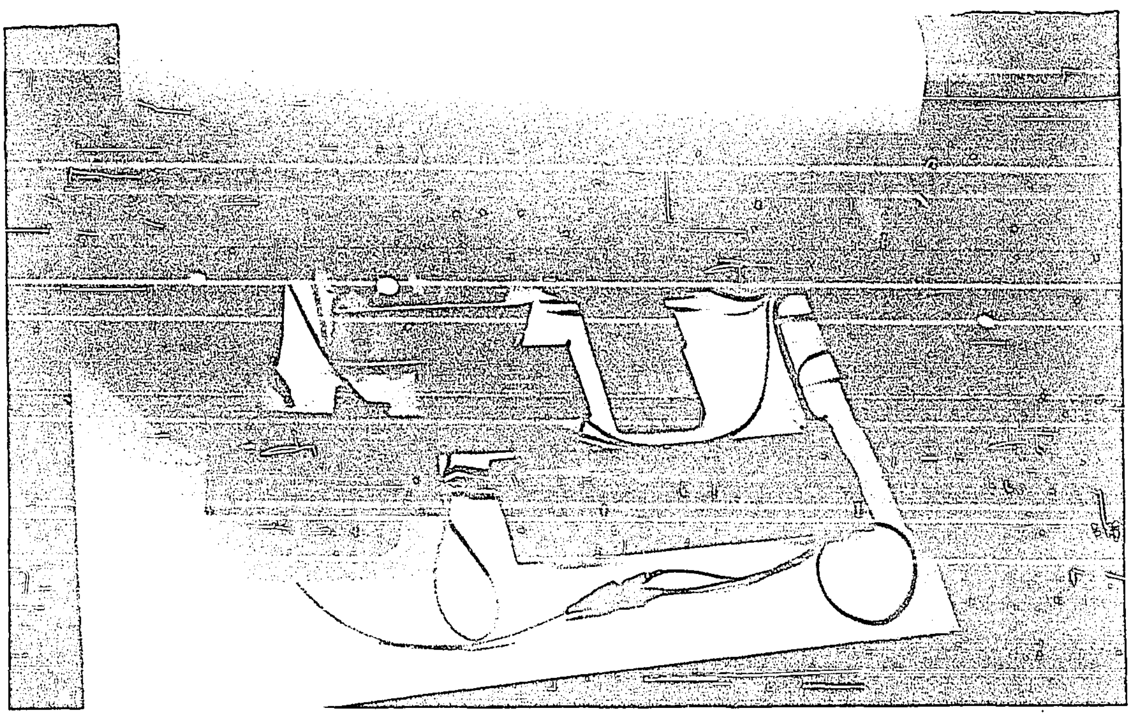
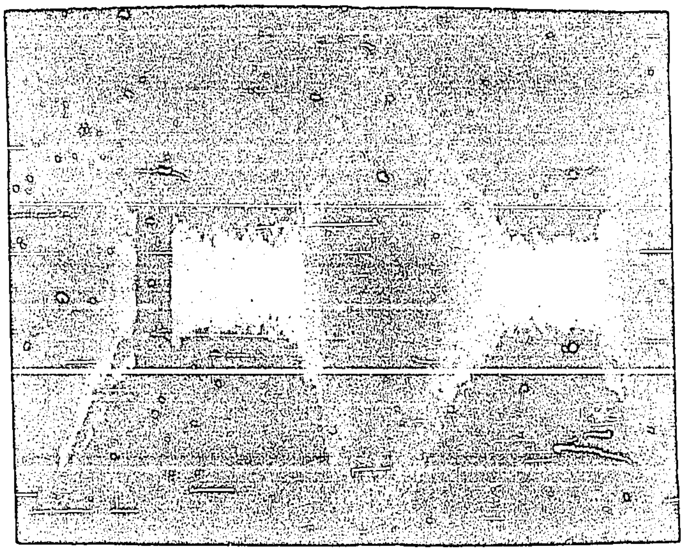


Fig. 33. Typical photo of a 2 second signal on a 60 cps transmission line after it has passed through a narrow band receiver with a signal-plus-noise to noise-power ratio of 16 to 1. The vertical scale is 50 millivolts per centimeter and the horizontal scale is 1 second per centimeter



CONCLUSIONS

The important aspects to consider when communication channels are made from a remote location to the source are the following:

(1) The method of generating the current signal that is sent to the source.

(2) The physical location of the current signaling equipment at the remote location.

(3) The method of detecting the current signal that is sent to the source and its physical location.

(4) The amount of current signal that is required for a given system.

First is the method of generating the current signal that is sent to the source. If a load is placed on the line at a remote location for precisely T_i seconds and removed from the line for precisely T_j seconds and repeated N times the current drawn by the load will have predictable frequency components in it. If the sum of T_i plus T_j is T_k the first useable frequency component for communicating with the source is $(60 + 1/T_k)$ cps. The communication channel is limited to the 70 to 110 cps range so the value of T_k is between 1/50 and 1/10 seconds. The values of T_i and T_j need not be the same. If the zero crossing of the 60 cps voltage is used as a timing reference and if T_k is an integer of 1/120 seconds the maximum time T_k could have is 12/120 seconds and the minimum time T_k could have is 3/120 seconds. This would assure that the first useable frequency component for communicating with the source will be in the 70 to 110 cps range.

Second is the physical location of the current signaling equipment at the remote location. The current signaling equipment may be placed at any location in the field. Its location is independent of the system because the current signal generated by it comes from the source. The load may be added to any one of the three phases and still its physical location is independent of the system.

Third is the method of detecting the current signal that is sent to the source and its physical location. Each power plant has one or more three-phase generators in parallel to supply the required power to its system. These generators have a common node and the power that is generated flows to several outgoing lines for distribution into the field. Each of these outgoing lines is monitored before leaving the power plant. Also each of these lines only supply a fraction of the total power that is generated. The physical location of the detecting equipment is placed at each of the several monitoring stations. This is done because the magnitude of the current signal required to communicate with the source is a minimum at this location and also the number of communication channels available is increased by the number of outgoing lines. A single receiver is required at each monitoring station if all the outgoing lines are used as a communication channel at the same time. A single receiver could be used for the entire power plant but this would imply that only one outgoing line could communicate with the source at any one time.

To detect the current signal on a three-phase system at a monitoring station a current transformer must be placed around all three outgoing conductors at the same time. This assures that no matter which phase the load is added to, the current signal must pass through the current transformer. The primary of this transformer is the three outgoing conductors. The secondary has a number of turns on it so that the secondary current is a fraction of the primary current. A very small resistor is placed across the secondary and left there before the transformer is placed around the three main conductors. The voltage drop across this resistor is proportional to the actual current that flows in the three conductors.

For a completely balanced three-phase system the 60 cps current in the current transformer will cancel. The noise currents on the line are assumed to be completely random so these noise currents do not cancel. If each conductor has 100 amperes of current flowing in it the noise produced by these three conductors is assumed to have the same value as a single conductor carrying 300 amperes. For a normal three-phase system a notch filter is required to attenuate the 60 cps component before the signal is fed into the narrow band receiver. This narrow band receiver must have good rejection characteristics and have a bandwidth in the order of one or two cycles.

Fourth is the amount of current signal that is required for a given system. Several things must be known about the communication channel before an approximation can be given. The approximate rms noise current for a given bandwidth and frequency must be known or measurements taken to determine it. The bandwidth of the receiver must be known, also the time required for each binary bit of information, and the signal-to-noise power ratio. The maximum line current that the system produces at 120 volts is also required. It is assumed that the load is added to and taken off a 120 volt line at the predetermined rate.

It is assumed that the average rms noise current per one cycle bandwidth is i_N for a conductor carrying 100 amperes. For a system that supplies 10,000 amperes of current at 120 volts the average rms noise current is $i_{NT} = (10{,}000/100)^{1/2}\ i_N$. The rms noise currents add as the square root because they are assumed to be completely random. The noise power, P_N, is proportional to i_{NT}^2. For a one cycle receiver the noise power is equal to i_{NT}^2. The frequency of the receiver is tuned to 75 cps and its bandwidth is one cycle. It is known that the time T_k must be equal to 1/15 seconds because the receiver is tuned to a frequency 15 cps above 60 cps. If a load is added to the line for two complete cycles and removed for two complete cycles the first useable frequency component would be at 75 cps. Assume that the binary bit rate is one bit per second. The receiver bandwidth is one cycle so the power contained in one cycle centered at 75 cps is found from the integral of two times the finite-time spectral density function from 74.5 to 75.5 cps. This value of power, P_s, is for a unit current. The actual power required, P_{sT} is equal to $A^2 P_s$ because P_s is for a unit current and P_{sT} is for a current of unknown amplitude, A. If a signal-to-noise power ratio of 15 to 1 is needed it is now possible to find the actual magnitude of the current required.

$$P_{sT} = A^2 P_s = 15 i_{NT}^2 \qquad (102)$$

$$A = (15/P_s)^{1/2} i_{NT}$$

For a one megawatt resistive system a predicted current of 6.5 amperes is all that is required to communicate with the source from any location in the system if the experimental results are used.

The important aspects to consider when communication channels are made from the source to any remote location in the field are the following:

(1) The method of generating the voltage signal that is sent to the remote locations in the field.

(2) The physical location of the voltage signaling equipment at the power plant.

(3) The method of detecting the voltage signal that is sent to the remote location and its physical location.

(4) The amount of voltage signal power that is required for a given system.

First is the method of generating the voltage signal that is sent to the remote locations in the field. In a three-phase system the voltage signal must be applied in series with each of the three-phase generators and each signal must be 120 degrees apart. This assures that the signal power will have the same distribution as the 60 cps power in the field.

The voltage signal is applied to each phase by using a current transformer, power amplifier, and a local oscillator. The current transformer is assumed to be in the shape of a doughnut that can be opened and clamped completely around a conductor. The primary of the current transformer is the power line conductor that is in the center of the current transformer. The secondary has several turns on it and the power amplifier or a known resistor is connected to the secondary before the current transformer is placed around the power line conductor. Three current transformers are required and one must be placed around each of the three power line conductors. Doing this assures that a voltage signal is added in series with each of the three-phase generators. Three power amplifiers or the equivalent are required to supply the needed power. A single local oscillator may be used but a phase-shifting network is needed. This assures that the three signals applied to the line are all 120 degrees apart. The local oscillator is operated in the 70 to 110 cps range.

Second is the physical location of the voltage signaling equipment at the power plant. The physical location of this equipment is the same as the current detecting equipment. The reasons for using this location are also the same as before.

Third is the method of detecting the voltage signal that is sent to the remote location and its physical location. If a large enough voltage signal is applied to a three-phase system it can be detected at any location in the field. A notch filter is used to attenuate the 60 cps component and a narrow band receiver is used to detect the signal. This narrow band receiver must have good rejection characteristics and have a bandwidth in the order of one or two cycles.

Fourth is the amount of voltage signal power that is required for a given system. Several things must be known about the communication channel before an approximation can be given. The approximate noise power, P_N, for a 120 volt line and for a given bandwidth must be known or measurements must be taken to determine it. Also the frequency at which communication is desired, the bandwidth of the receiver, the time required for each binary bit of information, the signal-to-noise power ratio, and the maximum input power, P_{1T}, to the system must be known.

It is assumed that the operating frequency is 90 cps, the receiver bandwidth is one cycle, the time required for each binary bit is one second, the signal-to-noise power ratio is 15 to 1, and the maximum input power, P_{1T}, is one megawatt. The noise power, P_N, is proportional to the average rms noise voltage squared per one cycle bandwidth, e_N^2, on a 120 volt line. If all the voltage signal power, P_s, were contained in one cycle bandwidth the power required for a signal-to-noise power ratio of 15 to 1 would be

$$P_s = 15 \, P_{1T}(e_N^2/(120)^2) \tag{103}$$

From the finite-time spectral density function of a unit voltage signal, y(t), that is on for one second it is seen that all the power is not contained in one cycle bandwidth. For the above case only 77 per cent of the voltage signal power is contained in the one cycle bandwidth.

The actual power, P_{sT}, required is

$$P_{sT} = 1.3\ P_s = 19.48\ P_{1T}(e_N^{\ 2}/(120)^2) \qquad (104)$$

For a one megawatt system the required predicted power, P_{sT}, is 5.85 milliwatts. This amount of signal power is all that is required to communicate with any location in the system if the experimental results are used.

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